

**Dear Handling Topic Editor (Yuntao Zhou) and Reviewers:**

**Re: essd-2026-341**

We sincerely appreciate your time and effort in reviewing our manuscript. Your critical comments and insightful suggestions have been invaluable in improving the quality and clarity of our work. We have carefully addressed all the concerns raised during this round of review. In the revised manuscript, all modifications made in response to the reviewers' comments are highlighted in blue for easy identification. Additional experiments, analyses, or clarifications have been incorporated as suggested. The manuscript has been thoroughly proofread to enhance its rigor and readability. Below, we provide point-by-point responses to the reviewers' specific questions. Should further revisions be required, we are committed to addressing them promptly.

**Reviewer 1**

**This manuscript uses a physics-informed deep learning approach to reconstruct a global daily sea surface ageostrophic current dataset for 1993-2023. The topic is meaningful, since many existing surface current products are mainly based on geostrophic currents, or geostrophic currents with Ekman corrections, while ageostrophic processes, especially nonlinear ageostrophic motions, are less well represented. The attempt to introduce momentum-balance constraints into a neural-network framework and generate a long-term global dataset is valuable.**

**Major Comments**

**Q1: In Section 4.1, the observational validation is based on only one TAO mooring site. This example is helpful, but its representativeness is limited. Additional observational sites from different regions (such as polar regions, equatorial regions, and coastal areas), or other independent observational data, would better demonstrate the applicability of the dataset under different dynamical regimes. Whether the errors exhibit systematic differences across different ocean regions should be clarified.**

**Response to Q1:**

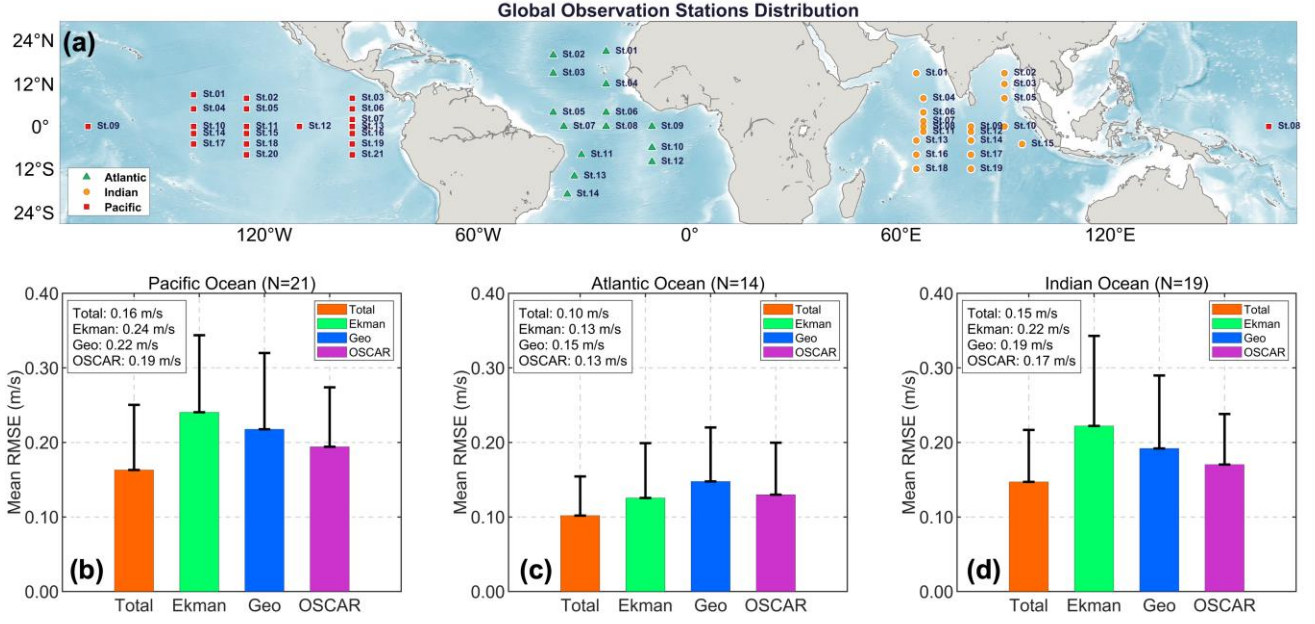
We sincerely thank the reviewer for this constructive comment. Following your suggestion, we have systematically expanded our validation against 54 in-situ observations from the TAO, PIRATA, and RAMA arrays (<https://climatedataguide.ucar.edu/climate-data/tropical-moored-buoy-system-tao-triton-pirata-rama-toga>). Besides, the widely used OSCAR dataset was also included in the comparison.

## 1. Overall error analysis and expanded validation

As shown in Figure R1(a), those 54 in-situ observations stations are widely distributed across the Pacific (21 stations), Atlantic (14 stations), and Indian Oceans (19 stations). Figure R1(b)–(d) display the Root Mean Square Error (RMSE) of the AC-PIDNN-corrected total currents, OSCAR currents, geostrophic currents (Geo), and Ekman corrected currents (Ekman) against the in-situ measurements. The statistical results are as follows:

- **Pacific Ocean:** The RMSE of the AC-PIDNN-corrected total current is 0.16 m/s, which outperforms both the Ekman corrected (0.24 m/s) and Geo (0.22 m/s) currents. Furthermore, compared to the standard OSCAR product's error of 0.19 m/s against in-situ data, the AC-PIDNN product reduces the error by approximately **15.8%**.
- **Indian Ocean:** The RMSE of our corrected current is 0.10 m/s, outperforming Ekman corrected (0.13 m/s) and Geo (0.15 m/s) currents. Compared to the OSCAR product's error of 0.13 m/s, the AC-PIDNN product reduces the error by approximately **23.1%**.
- **Atlantic Ocean:** The RMSE of our corrected current is 0.15 m/s, outperforming Ekman corrected (0.22 m/s) and Geo (0.17 m/s) currents. Compared to the OSCAR product's error of 0.17 m/s, the AC-PIDNN product reduces the error by approximately **11.8%**.

Overall, across all three major ocean basins, the corrected surface circulations are closer to the in-situ observations compared to the traditional geostrophic and Ekman corrected components, and they consistently demonstrate improved accuracy compared with OSCAR products.



**Figure R1.** (a) Spatial distribution of TAO/PIRATA/RAMA mooring sites across the Pacific, Atlantic, and Indian Oceans. In-situ observation stations in each ocean basin are individually numbered to facilitate subsequent comparative experiments. (b)–(c) Bar charts illustrating the averaged root-mean-square error (RMSE) between TAO/PIRATA/RAMA mooring observations, OSCAR products and the AC-PIDNN-corrected current fields, geostrophic currents, and Ekman corrected currents for the Pacific, Atlantic, and Indian Oceans, respectively.

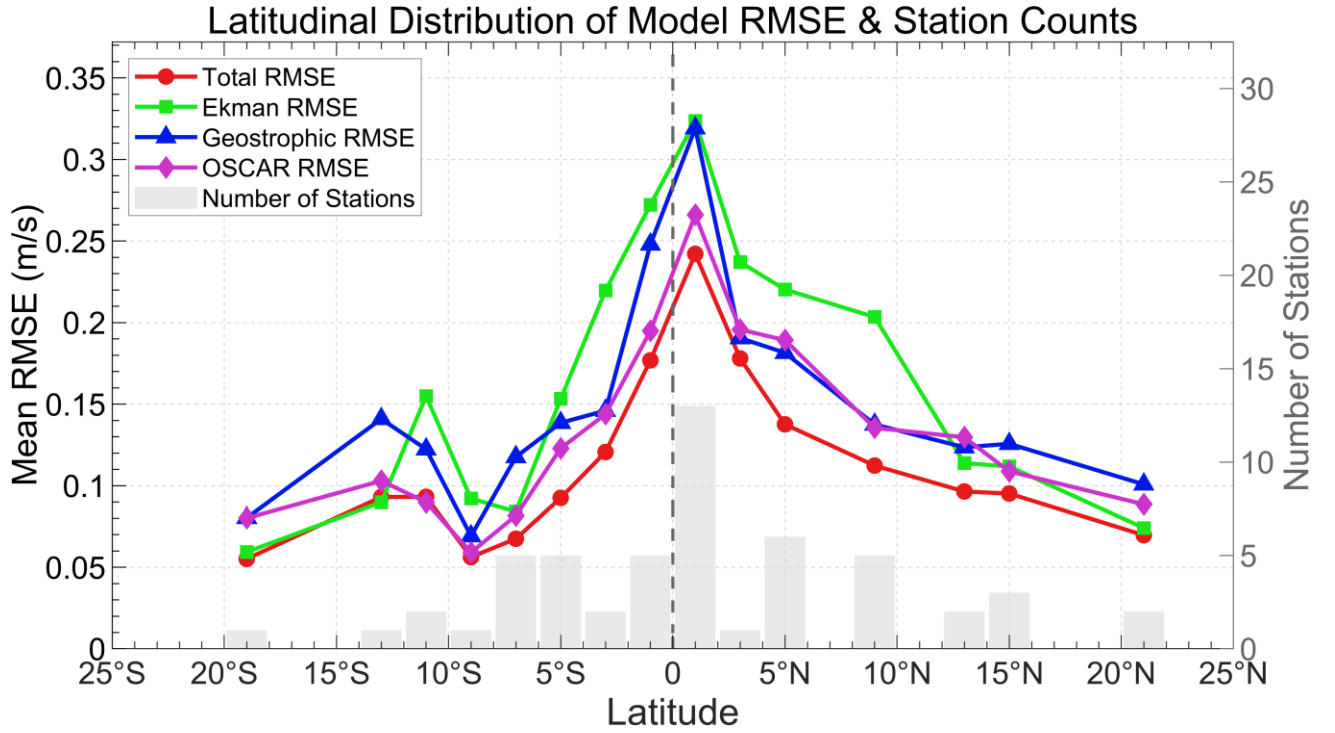
## 2. Error distribution characteristics and systematic differences

To address your query regarding whether the errors exhibit systematic differences across different ocean regions, we conducted an in-depth latitudinal analysis using the in-situ data, as shown in Figure R2. The results reveal that the dataset's error characteristics do exhibit spatial differences:

- Larger errors are primarily concentrated near the equator (within a latitude band of approximately  $\pm 5^\circ$ ).
- Conversely, accuracy significantly improves in mid-to-low latitudes, aligning much more closely with field observations. Specifically, the RMSE gradually decreases from 0.25 m/s near the equator down to 0.05 m/s in these higher-latitude observation areas.

Overall, within the  $\pm 25^\circ$  latitude range, the AC-PIDNN-corrected product consistently remains closer to the field observations than the OSCAR product. Regarding the valuable suggestion to include high-latitude, it is important to note that there is a lack of long-term continuous surface observations. We have explicitly

acknowledged this limitation regarding high-latitude applicability in the **section 4.1.2** of the revised manuscript. We once again thank you for your valuable and constructive comments.



**Figure R2** presents the latitudinal error curves of the in-situ measurements against both the corresponding AC-PIDNN-corrected currents and the OSCAR products across the 54 observation stations.

**Q2: Comparisons with existing mature surface current products, such as OSCAR, or other commonly used products, would strengthen the manuscript. At present, the manuscript mainly compares AC-PIDNN with geostrophic currents, Ekman-corrected currents, an iterative method, and ROMS simulations. However, readers will likely want to know how this new dataset differs from, or improves upon, existing surface current products. Such comparisons would help clarify the actual added value of the proposed dataset.**

**Respond Q2:**

We sincerely thank the reviewer for this constructive comment. Besides the comparison shown in Response 1, we further examined the circulation pattern and divergence with the OSCAR.

We compared the 30-year mean (1993–2022) total surface flow fields derived from OSCAR with those reconstructed by our method. Overall, the two datasets exhibit similar large-scale circulation patterns. However, because the PIDNN reconstruction additionally accounts for the nonlinear current component, its current magnitudes are generally larger than those in OSCAR and show better agreement with the observations (Figures R1–R2). The global mean total velocity for OSCAR is 0.16 m/s, compared to 0.19 m/s for AC-PIDNN. The differences are particularly pronounced in high-kinetic-energy western boundary current regions, such as the Kuroshio and Gulf Stream, equatorial region, and Southern Ocean. In these regions, AC-PIDNN more effectively reconstructs the strong currents and intense nonlinear advection characteristic of western boundary current systems.

The incorporation of nonlinear effects also leads to stronger surface divergence in AC-PIDNN than in OSCAR, although the two datasets retain broadly similar large-scale spatial patterns. As shown in **Figure R3**, the divergence field reconstructed by AC-PIDNN exhibits greater intensity and sharper spatial gradients, particularly in dynamically energetic regions. These differences indicate that AC-PIDNN provides an improved representation of multiscale surface-current dynamics and associated convergence–divergence structures. We have added these results to the Supplementary Material as Figure S3.

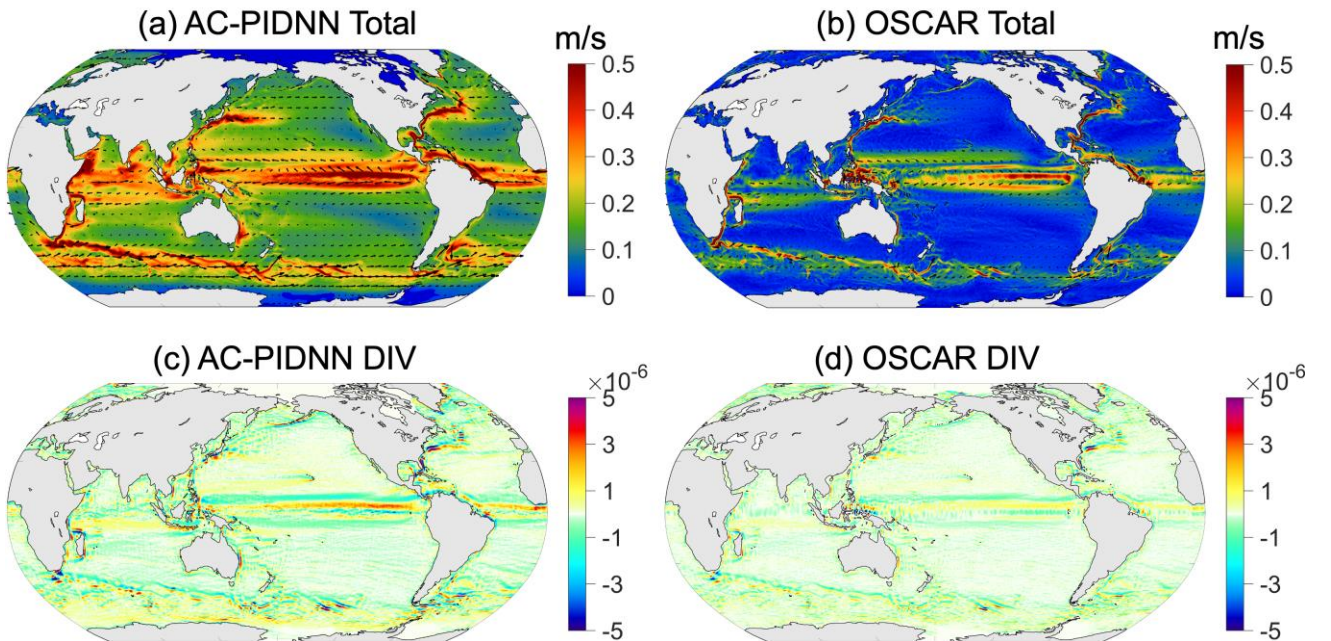


Figure R3. (a) 1993–2022 mean flow field from AC-PIDNN; (b) 1993–2022 mean flow field from the OSCAR product; (c) divergence of the 1993–2022 mean flow field from AC-PIDNN; (d) divergence of the 1993–2022 mean flow field from the OSCAR product.

**Q3: A simple ablation experiment would help show what the physics loss actually contributes. Since the first training stage already uses only  $Loss_{data}$ , the authors could simply save the Stage 1 output and use it as a data-loss-only baseline. Comparing this baseline with the final AC-PIDNN output using station-based observational errors and the Bal balance would make it clearer whether the later physics-informed stages provide a real improvement beyond the initial data fitting.**

**Respond Q3:**

We sincerely thank the reviewer for this insightful suggestion. To systematically address this, using the year 2021 as an example, we executed the suggested ablation experiment to examine the improvement by physics-informed stages. By comparing output from Stage 1, Stage 2 and Stage 3 against both theoretical momentum balance metrics (Bal) and station-based observational errors, we can clearly delineate the value added by the physical constraints beyond pure data-driven mapping.

### **1. Impact on Momentum Balance (Bal)**

**Figure R4(a)–(c)** illustrate the annual mean surface current fields generated by the AC-PIDNN model during Stages 1, 2, and 3, respectively. Correspondingly, **Figures R4(d)–(f)** map the momentum balance residual (Bal) for these three stages. The progression reveals a stark contrast in dynamical consistency:

- **Stage 1 (Data-Loss Only Baseline):** Because this initial stage relies exclusively on  $Loss_{data}$  without any embedded physical constraints, the output entirely lacks momentum balance correction, leading to severe dynamical inconsistencies where the momentum residuals exceed **100%** across the vast majority of the global ocean regions, as shown in **Figure R4 (a) and (d)**.
- **Stage 2 (Hybrid Loss Optimization):** Upon introducing a hybridized loss function that begins blending physical constraints with observational data, we observe a dramatic correction in the flow field's dynamical structure. The global mean momentum residual drops precipitously to approximately **51%**, as shown in **Figure R4 (b) and (e)**.
- **Stage 3 (Fully Physics-Constrained Final Output):** In the final stage, the output is rigorously constrained by the complete physics loss. This proper parameterization forces the reconstructed flow

field to obey the fundamental equations of motion, vastly improving the momentum balance and reducing the global average momentum residual to a mere **1.2%**, as shown in **Figure R4** (c) and (f).

This progression clearly demonstrates that the later physics-informed stages provide a profound improvement that extends well beyond initial data fitting. By enforcing momentum conservation on the purely data-driven output, the subsequent physics-informed stages produce a flow field that is both physically coherent and statistically accurate, thereby enhancing the hydrodynamic reliability of the reconstructed surface currents.

## **2. Validation Against In-Situ Observational Errors and OSCAS products**

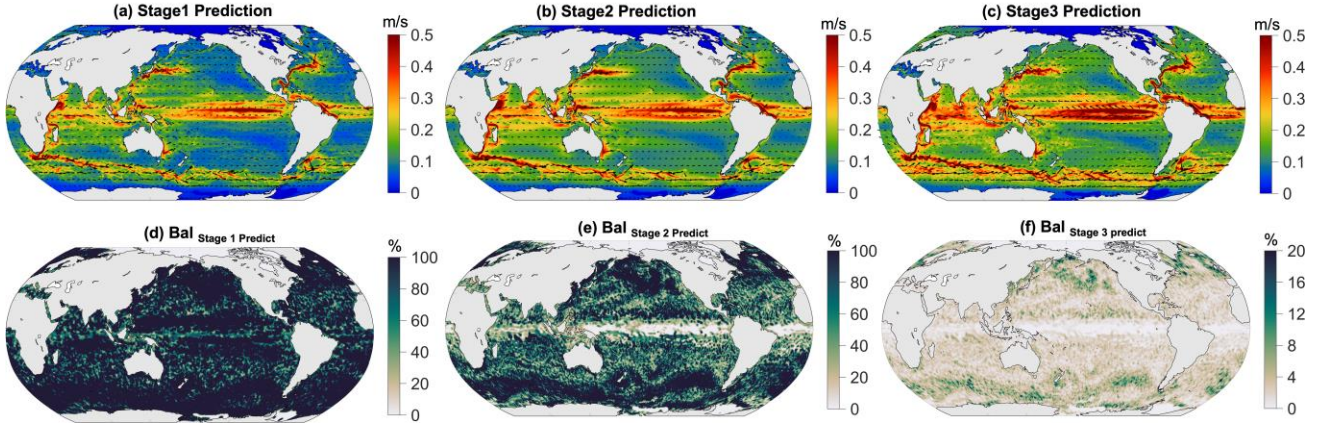
To confirm that this improved dynamical balance translates to real-world accuracy, we further evaluated the outputs of all three training stages, as well as the standard OSCAR product, against independent in-situ mooring data as shown in **Figure R5**. As expected, the progressive incorporation of physical laws systematically refines the accuracy of the modeled velocities:

- **Stage 1 Average Error:** 0.17 m/s
- **Stage 2 Average Error:** 0.16 m/s
- **Stage 3 Average Error:** 0.13 m/s
- **OSCAR Product Average Error:** 0.17 m/s

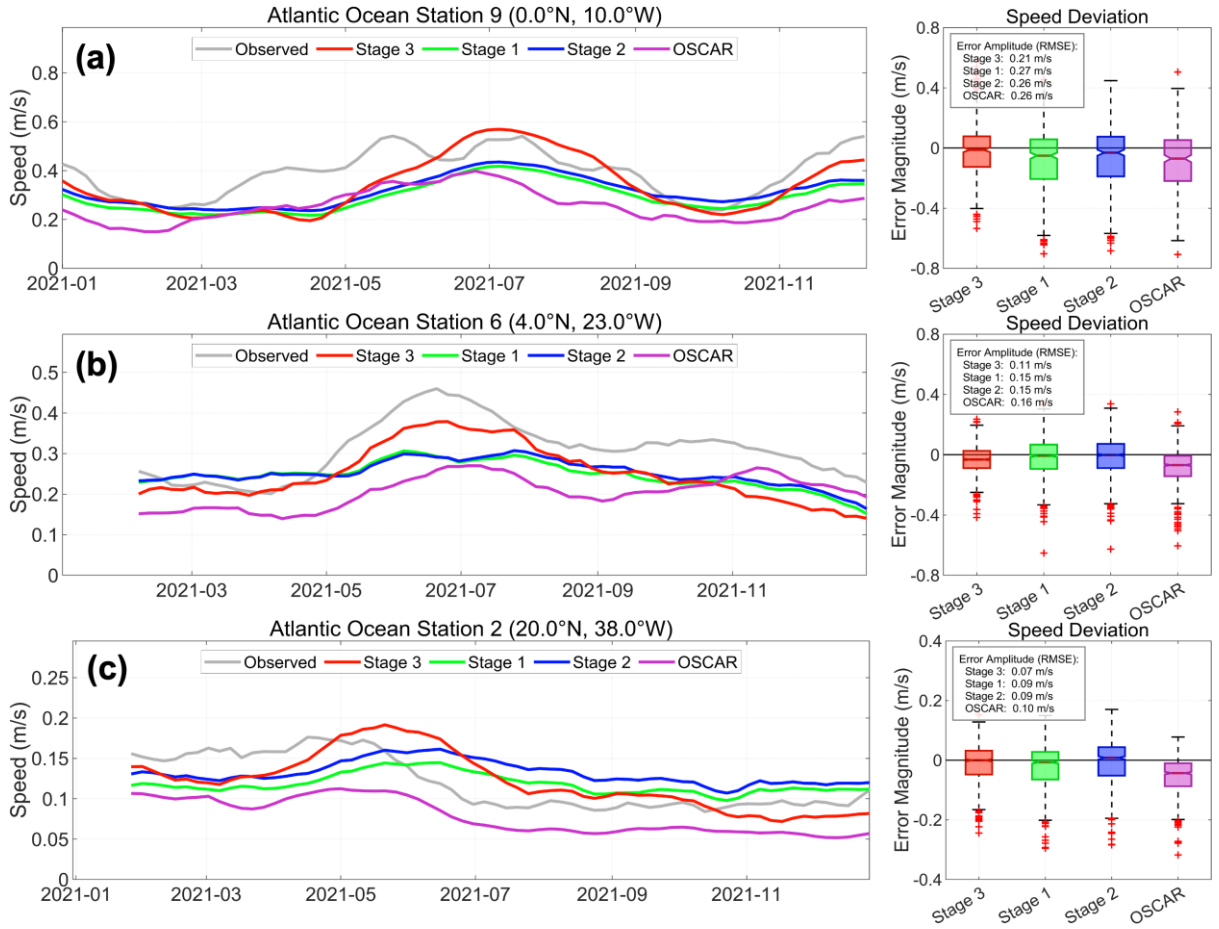
The data clearly shows that the Stage 3 output aligns most closely with the in-situ measurements, significantly outperforming the Stage 1 baseline, the intermediate Stage 2 output, and the OSCAR product.

This direct comparison with station-based observational errors firmly substantiates the superiority and absolute necessity of the physics-informed training stages. We have incorporated the comprehensive results of this ablation experiment, alongside the associated figures into the **Supplementary Material (Figure S1 to Figure S2)**. We once again thank you for your valuable and constructive comments.





**Figure R4.** (a) Mean output current field of AC-PIDNN Stage 1 in 2021; (b) Mean output current field of AC-PIDNN Stage 2 in 2021; (c) Final mean output current field of AC-PIDNN Stage 3 in 2021; (d) Momentum balance (Bal) results for Stage 1; (e) Momentum balance (Bal) results for Stage 2; (f) Momentum balance (Bal) results for Stage 3.



**Figure R5.** Comparison of the output current fields from the three training stages of AC-PIDNN and OSCAR product velocities against in-situ observational data.



**Q4: The data upload method on the Zenodo website may not meet the requirements of ESSD. Data must be submitted in a more appropriate manner and additional information must be provided regarding the format of the data set, file naming conventions, variable units, missing values, metadata, and whether the NetCDF files comply with the CF standard.**

**Respond:**

We sincerely thank the reviewer for highlighting these critical omissions. We fully agree that comprehensive metadata and strict adherence to data standards are essential for the usability and longevity of the dataset. We have thoroughly revised our Zenodo repository and the dataset's internal documentation to align with ESSD requirements. The updated repository is now available at: DOI: 10.5281/zenodo.19966716. Specifically, we have addressed the reviewer's concerns as follows:

**1. Data Upload Method and Accessibility Data Upload Method and Accessibility**

Due to the comprehensive 31-year span and high spatial resolution ( $0.25^\circ$ ) of our dataset, the total data volume exceeds 500 GB. Because this vastly surpasses Zenodo's standard per-upload storage limits, hosting the entire dataset directly on Zenodo in a single upload is technically unfeasible. To provide a practical and compliant solution, we have adopted a hybrid access approach:

- We have uploaded a representative sample .nc file directly to the Zenodo repository so that users can immediately inspect the data structure, metadata, and variables.
- We have included a comprehensive README.txt file on Zenodo. This document details the exact directory structure and provides direct, high-speed download links to the complete 500 GB dataset hosted on a dedicated S3 server (<https://s3.jn1.is.shanhe.com/umdr2026/>).

**2. CF Standard Compliance and Metadata Enhancements CF Standard Compliance and Metadata Enhancements**

We have explicitly updated the Zenodo description and ensured our NetCDF files comply with the Climate and Forecast (CF) Metadata Conventions (CF-1.8). The repository page now clearly documents the following details:

- Data Format: NetCDF-4 Classic (netcdf4\_classic).
- File Naming Conventions: Detailed the standardized directory and file naming structure ([YYYY]/[MM]/[YYYY]\_[MM]\_[DD].nc).
- Variable Details & Units: Provided a comprehensive table defining every variable (e.g., ugeo, uekman, uadv, utotal), including their standard long names, dimensions, and strict inclusion of scientific units (e.g., m/s, m/s<sup>2</sup>, m).
- Missing Values: Clarified the handling of missing data (e.g., over landmasses or un-computable grid points) by explicitly defining the use of the standard \_FillValue = NaN indicator.
- Global Metadata: Outlined the inclusion of global attributes to track provenance, including creation\_date, author, and Conventions.

We believe these updates provide the clarity, transparency, and standardization required by ESSD, ensuring the data is highly accessible and easily interpretable for future researchers.

### Specific Comments

**Q5: The method description is not yet sufficiently clear, especially regarding the training labels. Based on the code provided by the authors, the input variables described in the manuscript do not seem to fully match the inputs used in the code. Also, the training labels in the code appear to be a pseudo-target obtained by adding the geostrophic current and the Ekman current, rather than an independently observed total current. This approach may be reasonable, but it should be stated clearly in the method section. Otherwise, it is hard to understand how the model moves from a geostrophic + Ekman baseline toward a nonlinear ageostrophic correction.**

### Respond:

We sincerely thank the reviewer for this meticulous and insightful observation. We fully agree that the original description of the model's inputs and training labels lacked the necessary clarity and transparency. We have carefully revised the methodology section to ensure strict alignment between the manuscript and the provided codebase, and to explicitly explain the rationale behind our training strategy.

Specifically, we have made the following modifications in the revised manuscript:

### **(1) Alignment of Input Variables**

As the reviewer correctly pointed out, there was a discrepancy between the text and the actual code. We have thoroughly reviewed and rewritten the relevant paragraphs in the Methodology section (see **Section 3 in the revised manuscript**) to comprehensively list and define all input variables. The updated text now strictly mirrors the input features explicitly used in our deep learning code, eliminating any ambiguity.

### **(2) Clarification of the "Pseudo-Target" Training Labels**

We highly appreciate the reviewer's acute observation regarding the training labels. We have now explicitly stated in the revised manuscript (see **Section 3.2.3 of the revised manuscript**) that the training labels are indeed "pseudo-targets" derived from the superposition of the geostrophic current and the Ekman corrected current, rather than dense, independent real-world observations (such as drifter buoy data).

### **(3) Explanation of the Learning Mechanism for the Ageostrophic Correction**

To address the reviewer's valid concern about how the model learns the nonlinear ageostrophic correction from this baseline, we have added a dedicated explanation of our physical and algorithmic logic:

- We clarify that because high-resolution, global, and independent observations of total sea surface currents are too sparse to support AI model training, we employ the physics-derived baseline (geostrophic + Ekman) as the primary training target.
- We further elaborate on how the model utilizes these pseudo-targets in conjunction with the designed loss functions and physical constraints (e.g., incorporating the nonlinear advective terms and momentum residuals) to move beyond the linear baseline. This approach forces the model to capture and output the non-linear ageostrophic corrections necessary to bridge the gap between the simplified linear dynamics and the actual complex flow field.

We believe these revisions directly address your concerns, significantly improving the methodological transparency, reproducibility, and physical interpretability of our model (**please refer to Section 3.2.3 in the revised manuscript and Figure S1 in the supplementary material**). Thank you again for pointing out this critical detail.

**Q6: The manuscript mentions an activation function, but it does not specify which activation function was actually used. The final model settings should be included in the manuscript.**

**Respond:**

We thank the reviewer for pointing out this omission. We agree that specifying the exact activation function and hyperparameters is crucial for the reproducibility of our deep learning model. In the Supplementary Material, we have added Table R1 to explicitly detail the activation functions and provide a comprehensive summary of the final model configurations. Specifically, the following details have been added:

- **Activation Function:** We have explicitly stated that the **tanh** activation function was utilized in the hidden layers of our network because of its efficiency in mitigating vanishing gradients and accelerating convergence during training.
- **Final Model Settings:** To present the model configurations transparently, we have compiled all hyperparameters into a table, as shown in Table R1 (**Table S1** in the Supplementary Material). We believe these additions fully resolve the ambiguity and ensure that our model architecture is fully transparent and reproducible.

We have detailed these revisions in Lines 210 of the revised manuscript and the accompanying **Supplementary Material Table S1**.

**Table R1. Hyperparameters and training configurations of the AC-PIDNN model.**

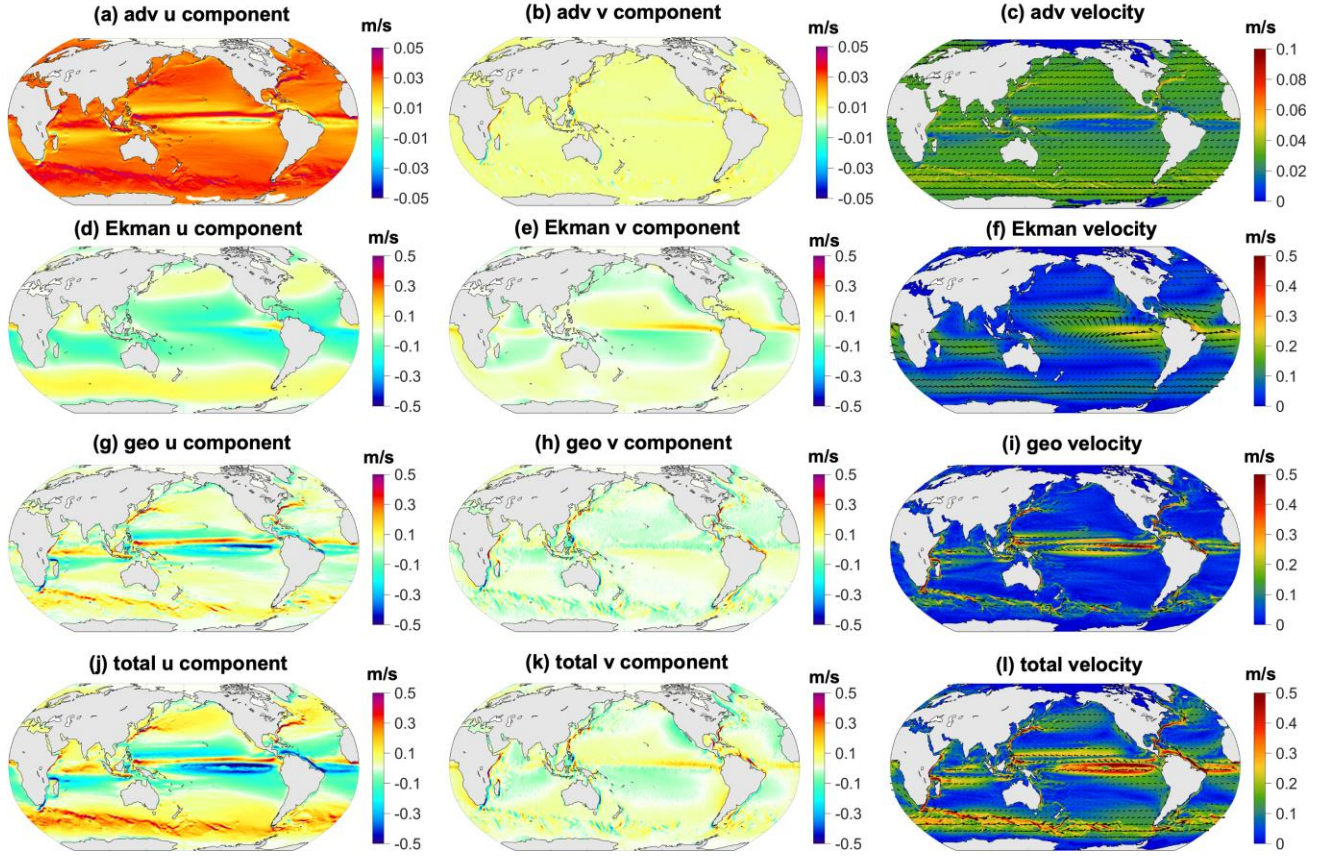
| Category                                                     | Hyperparameter / Setting               | Value / Description |
|--------------------------------------------------------------|----------------------------------------|---------------------|
| Network Configuration                                        | Optimizer                              | Adam                |
|                                                              | Weight Initialization                  | Xavier              |
|                                                              | Activation Function                    | Tanh                |
| Regularization (to prevent overfitting)                      | Dropout Rate                           | 0.3                 |
|                                                              | L2 Regularization Factor ( $\lambda$ ) | 0.01                |
| Stage 1: Initialization<br>(Quickly fit geostrophic current) | Loss Function                          | $L_{data}$          |
|                                                              | Learning Rate                          | 0.001               |
|                                                              | Epochs                                 | 1000                |
|                                                              | Loss Function                          | $L_{hybrid}$        |

|                                                                        |                        |                      |
|------------------------------------------------------------------------|------------------------|----------------------|
| Stage 2: Transition<br>(Gradually introduce<br>physical constraints)   | Hybrid Loss Proportion | 1:1                  |
|                                                                        | Learning Rate          | 0.0001               |
|                                                                        | Epochs                 | 1000                 |
| Stage 3: Physics-Informed<br>(Minimize momentum<br>equation residuals) | Loss Function          | $L_{\text{physics}}$ |
|                                                                        | Learning Rate          | 0.00001              |
|                                                                        | Epochs                 | 10000                |

**Q7: In Figure 4(a-c), the nonlinear advective component is relatively weak, and under the current color scale its main spatial patterns are difficult to see by eye.**

**Respond:**

We gratefully acknowledge the reviewer for pointing out this visual clarity issue. To address this, we have regenerated **Figure 4 as shown in Figure R9**. Specifically, we have applied an independent, narrower color scale (adjusted colorbar limits) for the nonlinear advective component in panels (a-c). This adjustment significantly enhances the visual contrast, making the spatial patterns and localized variations of the advective term particularly its intensified signals around dynamically active regions such as western boundary currents and mesoscale eddies clearly visible to the reader. The updated Figure 4, along with a revised caption explaining the distinct color scales, has been incorporated into the revised manuscript.



**Figure R9** Results of Sea Surface Circulation Correction from 1993 to 2023, using the AC-PIDNN model, incorporating geostrophic, nonlinear advective and Ekman corrected components. Panels (a)–(c) show the u-component, v-component, and flow field of the nonlinear advective component; Panels (d)–(f) show the u-component, v-component, and flow field of the Ekman corrected component; Panels (g)–(i) show the u-component, v-component, and flow field of the geostrophic component; Panels (J)–(L) show the u-component, v-component, and flow field of the total circulation.

**Q8:** Line 157 contains “Error! Reference source not found.” This should be fixed.

**Respond Q8:**

We appreciate the reviewer pointing out this oversight. The broken cross-reference has been corrected to refer to **Line 155** in the revised manuscript. We have also verified all other citations and cross-references throughout the document to ensure their accuracy.



**Q9: The Figure 3 caption contains the phrase “the second column and column,” which appears to be incorrect.**

**Respond Q9:**

We thank the reviewer for their careful reading and for catching this typographical error. We have corrected the phrase in the caption of **Figure 3 (Figure 5 in revised manuscript)**. The text has been revised from “the second column and column” to “the second column” to accurately describe the corresponding panels in the figure. We have also carefully proofread all other figure captions in the manuscript to ensure accuracy and readability.

**Q10: Around line 430, “Guanglinag Liu” appears to be a spelling error.**

**Respond Q10:**

We sincerely thank the reviewer for their extremely careful reading of our manuscript and for catching this typographical error. We have corrected the spelling from “Guanglinag Liu” to “**Guangliang Liu**” in the revised manuscript. Additionally, we have conducted a thorough proofreading of the entire text and the reference list to ensure that all names and terminologies are spelled correctly.

# Daily Global Sea Surface Ageostrophic Current Dataset from 1993–2023 via Physics-Informed Deep Learning

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## Abstract

Surface ocean circulation shapes climate, air-sea exchange, and the transport of heat, carbon, and other tracers, yet most long-term global datasets rely primarily on geostrophic balance and therefore miss important ageostrophic motions. Here, we reconstruct global daily sea surface circulation at  $0.25^\circ$  resolution from 1993 to 2023 using a physics-informed framework that integrates satellite altimetry with momentum constraints from geostrophic balance, wind stress, and nonlinear advection. The resulting dataset is dynamically consistent, with a mean momentum-balance error below 1.5%, and reduces the median velocity error against independent mooring observations to  $0.10 \text{ m s}^{-1}$ , compared with  $0.20 \text{ m s}^{-1}$  for geostrophic currents and  $0.18 \text{ m s}^{-1}$  for Ekman-corrected currents. We show that geostrophic flow sets the large-scale circulation, whereas Ekman and nonlinear contributions are smaller but comparable in magnitude to each other. Although nonlinear advection contributes little to relative vorticity, it strongly shapes surface divergence and the fine-scale structure of eddy kinetic energy. Our results show that nonlinear ageostrophic flow is an essential component of global surface circulation and that neglecting it limits our ability to resolve surface transport and variability on climatically relevant scales.

Index Term: Ageostrophic Circulation, Ekman Dynamics, Physics-Informed Dense Neural Network, Satellite Altimetry

## 1 Introduction

Surface ocean circulation regulates climate, air-sea exchange, and the transport of heat, carbon, nutrients, and other tracers.

35 Over the past several decades, satellite remote sensing has provided the principal observational basis for studying upper-ocean circulation (Du et al., 2021; Fan et al., 2025; Shang et al., 2025). Geostrophic currents derived from sea surface height have become central to analyses of ocean dynamics (Xie et al., 2024), climate change, and ecosystem assessments (Timmermans and Marshall, 2020; Srinivasan and Tsontos, 2023). However, geostrophic currents neglect the influence of ageostrophic processes, such as wind stress and nonlinear effects (Hiron et al., 2021). Although ageostrophic processes are relatively small  
40 in magnitude, both observational data and numerical simulations have consistently shown that they substantially modulate ocean circulation (Rio et al., 2014; Edwards et al., 2024). Ageostrophic motions play a crucial role in the evolution and adjustment of ocean circulation. For example, nonlinear effects contribute to the evolution of mesoscale eddies (Li et al., 2022) and are important for the transport of heat, dissolved carbon, and tracers (Ueno et al., 2023). Studies have also shown that nonlinear processes can reduce geostrophic current speeds by more than  $0.2 \text{ m s}^{-1}$  (Cao et al., 2023b), whereas wind-driven  
45 Ekman effects can enhance sea surface current velocities (Rio et al., 2014). Neglecting ageostrophic processes (Armitage et al., 2017; Shankar et al., 2002; Cho et al., 2025) omits the complex features introduced by variations in nonlinear forcing and wind stress (Andres et al., 2008; Buckingham et al., 2021), leading to substantial discrepancies with drifter-observed velocities (Douglass and Richman, 2015; Uchida and Imawaki, 2003). This omission introduces uncertainties in understanding the long-term evolution and energy transformation of ocean circulation.

50 The influence of ageostrophic motions is especially pronounced in energetically active regions such as western boundary currents, the subtropical countercurrent, and the Antarctic Circumpolar Current, where their kinetic energy can be comparable to that of geostrophic motions (Zhang et al., 2019). The development of ageostrophic motions can disrupt geostrophic balance, promote forward energy transfer, and influence the inverse transfer of geostrophic energy, leading to an increase in the mean geostrophic wavenumber (Kafiabad et al., 2019; Thomas and Daniel, 2021). Ageostrophic motions are also closely linked to  
55 marine ecosystems: the intense vertical motions they induce provide an important pathway for delivering nutrients into the euphotic zone and thereby help regulate primary productivity and carbon sequestration in the global ocean (Aluie et al., 2018; Liu et al., 2025; Masuda et al., 2021).

Most existing corrections to sea surface circulation adopt a steady-state framework (Fraser et al., 2024), and two main methods are commonly used to represent nonlinear effects. One assumes axisymmetric vortices and reduces nonlinear forcing  
60 to the centrifugal term in the gradient wind equation (Ioannou et al., 2019). The other uses a perturbation expansion in which the nonlinear term is approximated by the rate of change of geostrophic velocity (Penven et al., 2014). In practice, most previous studies have used iterative schemes for circulation correction (Cao et al., 2023a; Penven et al., 2014). The iterative solution of the gradient wind equation was first developed in atmospheric science (Cao et al., 2023a; Penven et al., 2014) and later introduced into oceanography. Such schemes have been applied to a large gyre in the Mozambique Channel, mesoscale  
65 eddies in the Mediterranean Sea (Cao et al., 2023a; Penven et al., 2014; Morales-Márquez et al., 2023), and the correction of

the global sea surface circulation field (Cao et al., 2023b; Ruiz et al., 2014). However, these schemes can diverge when the gradient wind equation has no solution (Douglass and Richman, 2015; Qiu et al., 2019). Furthermore, the stopping criterion for these schemes is typically when the velocity residual either begins to diverge or decreases below a predefined threshold (Douglass and Richman, 2015; Qiu et al., 2019), which is commonly set to  $0.01 \text{ m s}^{-1}$  in the literature (Cao et al., 2023a; Arnason et al., 1962; Penven et al., 2014). As a result, the dynamical consistency of the corrected solution remains uncertain.

Recent advances in artificial intelligence (AI) have created new opportunities for oceanographic data analysis and reconstruction (Li et al., 2020; Dong et al., 2022; Qin et al., 2023; Cui et al., 2025). AI has been applied to infer ocean surface dynamic parameters and structures (Bonino et al., 2024; Jang et al., 2022; Lenain et al., 2026; Cui et al., 2026), such as subsurface eddy kinetic energy (Xie et al., 2024), subsurface thermohaline structure (Zhou et al., 2024), and sea surface temperature (Bonino et al., 2024; Kumar et al., 2021; Kartal, 2023). AI-based research on sea surface circulation is also beginning to emerge. Existing work primarily uses machine learning to replace traditional geostrophic relationships by combining satellite-observed variables, such as sea surface height, sea surface temperature, and wind stress, to reconstruct global sea surface circulation fields (Sinha and Abernathy, 2021; Manucharyan et al., 2021). However, these approaches do not fully integrate the governing physical equations of ocean surface circulation into the learning framework for ageostrophic correction. As a result, the reconstructed flow fields can retain non-negligible dynamical imbalances, as reflected in residual terms of the momentum equations.

Here, we develop a physics-informed dense neural network with ageostrophic circulation correction (AC-PIDNN) to reconstruct global sea surface circulation while explicitly enforcing momentum balance. The model integrates constraints from geostrophic balance, wind stress, and nonlinear advection to recover ageostrophic flow from satellite-observed sea surface height gradients. Using the AC-PIDNN, we generate a daily global ageostrophic current dataset from 1993 to 2023 at a spatial resolution of  $0.25^\circ$ . This framework enables a dynamically consistent assessment of how ageostrophic processes shape global surface circulation, divergence, and eddy energetics. The structure of this paper is as follows: Section 2 describes the data used; Section 3 introduces the proposed methodology; Section 4 presents the experimental results and discussions; Sections 5 and 6 present the code and data availability and conclusions, respectively.

## 2 Data

### 2.1 Satellite Data

The dataset used in this study includes the daily Absolute Dynamic Topography (ADT) gridded data, geostrophic currents  $\vec{V}_{geo} = (u_{geo}, v_{geo})$ , and the corresponding surface Ekman components of the global ocean, which were downloaded from <https://data.marine.copernicus.eu/product>. The ADT gridded data are provided by the French Space Agency (CNES) through the Archiving, Validation, and Interpretation of Satellite Oceanographic data (AVISO) service, and are processed by the CNES/CLS Data Unification and Altimeter Combination System (DUACS). This system integrates observations from all

Sentinel altimetry missions as well as other collaborative or opportunistic missions. The datasets incorporate high-quality altimeter measurements and geophysical corrections and are generated using a consistent correction system to ensure maximum data quality and to minimize temporal artifacts. This dataset also provides geostrophic current  $\vec{V}_{geo}$  based on the geostrophic balance. These data used in this study have a temporal resolution of 1 day and a spatial resolution of  $1/4^\circ$ .

The surface Ekman components are obtained from the global Copernicus Ocean (GLOBCURRENT) surface and 15-meter Ekman and geostrophic current datasets. These data are derived by applying an empirical Ekman model to ERA5 wind stress fields (Mulet et al., 2021; Rio et al., 2014), with the calculation taking into account both wind stress and the mixed layer depth. The inclusion of mixed layer depth reflects the dynamic variability of Ekman depth across time and space. This product provides datasets on a  $1/4^\circ$  regular grid, offering current velocity fields at the surface and at 15 m depth with hourly, daily, and monthly temporal resolutions.

## 2.2 Simulation Data

To validate the performance of the method proposed in this study, we conducted a high-resolution numerical simulation using the Regional Ocean Modeling System (ROMS) (Shchepetkin and McWilliams, 2005). The simulation results served as the “ground truth” for assessing the accuracy of our approach. ROMS is a widely used, high-resolution numerical framework designed for simulating regional-scale oceanic and atmospheric processes. By solving the governing equations of fluid motion, ROMS provides accurate simulations of ocean circulation, temperature, salinity, and other essential physical properties while maintaining strong dynamical consistency and numerical stability. In this study, the simulation configuration covers the western Pacific region ( $5^\circ\text{N}$ – $32^\circ\text{N}$ ,  $109^\circ\text{E}$ – $141^\circ\text{E}$ ) with a spatial resolution of 3 km. The output is provided as a 30-day mean field. This high-resolution setup effectively resolves mesoscale and submesoscale oceanic dynamical processes, thereby establishing a reliable and physically consistent numerical benchmark for evaluating the accuracy and effectiveness of our method. It also forms a foundation for further investigation of regional circulation characteristics and underlying dynamical mechanisms.

## 3 Methodology

### 3.1 The Steady State Momentum Balance Equations

In this study, the surface sea currents within the Ekman layer are assumed satisfy the steady state momentum balance between nonlinear advection, pressure gradient force, Coriolis force and wind stress (Sudre et al., 2013; Van Meurs and Niiler, 1997):

$$\overbrace{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}}^{ADV_x} = \overbrace{\widetilde{f}v}^{COR_x} - \overbrace{g \frac{\partial \eta}{\partial x}}^{PGF_x} + \overbrace{\frac{1}{h_e} \left( \frac{\tau_x}{\rho} - r_e u_{ekman} \right)}^{WIND_x} \quad (1)$$



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$$\overbrace{u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y}}^{ADV_y} = \overbrace{-fu - g \frac{\partial \eta}{\partial y}}^{COR_y \overbrace{PGF_y}} + \overbrace{\frac{1}{h_e} \left( \frac{\tau_y}{\rho} - r_e v_{ekman} \right)}^{WIND_y} \quad (2)$$

where,  $u$  and  $v$  are the zonal and meridional velocities;  $\eta$  is the sea surface height from ADT data;  $f$  represents the Coriolis force parameter;  $g$  is gravity acceleration; and  $\tau(\tau_x, \tau_y)$  is the wind stress field. The  $h_e$  and  $r_e$  are thickness of the Ekman layer and the linear drag coefficient that represents the vertical viscosity terms as a body force on the Ekman components. With the information from the locally estimated  $h_e$  and  $r_e$ , the  $\vec{V}_{ekman} = (u_{ekman}, v_{ekman})$ , can be calculated based on the wind stress

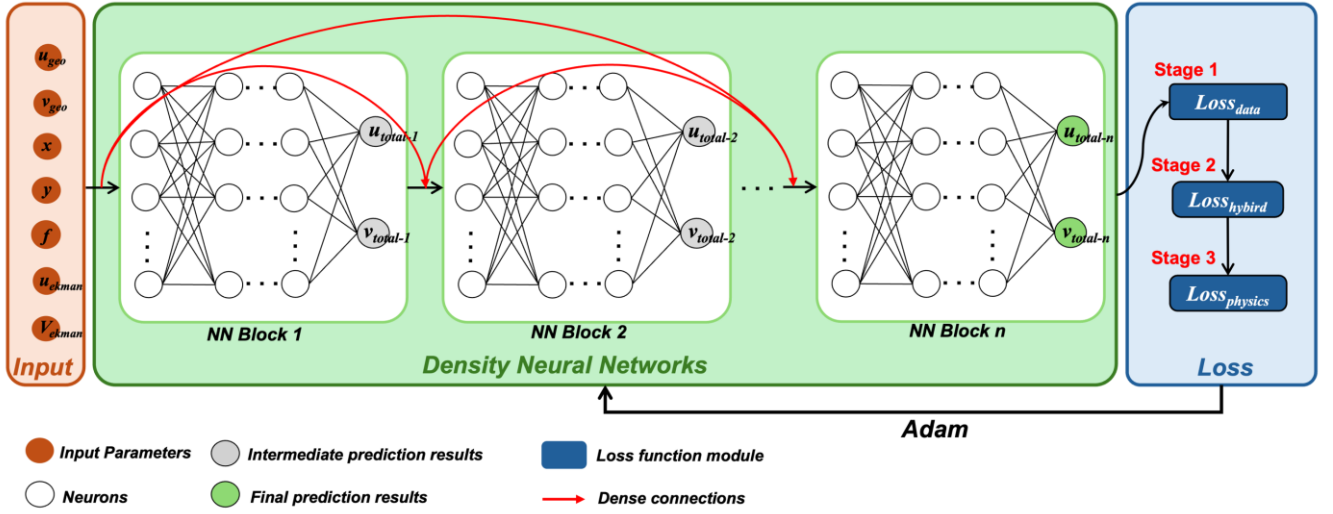
130

field. In our calculation, the  $\overbrace{\frac{1}{h_e} \left( \frac{\tau_x}{\rho} - r_e u_{ekman} \right)}^{WIND_x}$  and  $\overbrace{\frac{1}{h_e} \left( \frac{\tau_y}{\rho} - r_e v_{ekman} \right)}^{WIND_y}$  are represented by the  $-f v_{ekman}$  and  $f u_{ekman}$ , in which  $u_{ekman}$  and  $v_{ekman}$  are the Ekman component of current in GLOBCURRENT data to simplify the calculation.

### 3.2 Physics-informed Density Neural Networks for Correcting Ageostrophic Current (AC-PIDNN)

In this study, we propose a physics-informed neural network (PINN)-based architecture for correcting ageostrophic current, termed AC-PIDNN. The overall structure, as shown in Figure 1, consists of four components: input module, custom  
135 density neural networks module, and loss function module.

## Ageostrophic Current-Physics-informed Density Neural Networks (AC-PIDNN)



### Loss function in three stages of training

$$\begin{aligned}
 \text{Stage 1} \quad Loss_{data} &= \frac{1}{N} \sum_{i=1}^N \left| \overbrace{(u_{geo} + u_{ekman})}^{\text{pseudo-labels}} - u_{total-i} \right|^2 + \left| \overbrace{(v_{geo} + v_{ekman})}^{\text{pseudo-labels}} - v_{total-i} \right|^2 \\
 \text{Stage 2} \quad Loss_{hybrid} &= \frac{1}{N} \sum_{i=1}^N |ADV_x + COR_x + PGF_x + WIND_x|^2 + |ADV_y + COR_y + PGF_y + WIND_y|^2 + \lambda \times \frac{1}{N} \sum_{i=1}^N \left| \overbrace{(u_{geo} + u_{ekman})}^{\text{pseudo-labels}} - u_{total-i} \right|^2 + \left| \overbrace{(v_{geo} + v_{ekman})}^{\text{pseudo-labels}} - v_{total-i} \right|^2 \\
 \text{Stage 3} \quad Loss_{physics} &= \frac{1}{N} \sum_{i=1}^N |ADV_x + COR_x + PGF_x + WIND_x|^2 + |ADV_y + COR_y + PGF_y + WIND_y|^2
 \end{aligned}$$

Figure 1: Schematic diagram of the proposed sea surface circulation correction model (AC-PIDNN).

### 3.2.1 Input Module

The model inputs include geostrophic current components derived from the geostrophic approximation ( $u_{geo}, v_{geo}$ ), surface Ekman current components ( $u_{ekman}, v_{ekman}$ ), Coriolis parameter  $f$ , and geographical coordinates ( $x, y$ ). The input features be represented as:

$$\mathbf{X} = [u_{geo}, v_{geo}, x, y, f, u_{ekman}, v_{ekman}] \quad (3)$$

The geostrophic currents are calculated based on the geostrophic balance:

$$f v_{geo} - g \frac{\partial \eta}{\partial x} = 0 \quad (4)$$

$$-f u_{geo} - g \frac{\partial \eta}{\partial y} = 0 \quad (5)$$

In this study,  $u_{geo}$  and  $v_{geo}$  are directly derived from the AVISO dataset. However, Equations (4) and (5) become singular at the equator ( $f = 0$ ), and they are also problematic in its vicinity due to the weak Coriolis effect. To address this issue, within

the latitude band of  $\pm 2^\circ$ , this study adopts the so-called “*semi-geostrophic*” approximation to compute the equatorial current components  $\vec{V}_{sg} = (u_{sg}, v_{sg})$ , which replace the geostrophic currents in this region (Sudre et al., 2013):

150

$$u_{sg} = -\frac{g}{\beta} \frac{\partial^2 \eta}{\partial y^2} \quad (6)$$

$$v_{sg} = \frac{g}{\beta} \frac{\partial^2 \eta}{\partial x^2} \quad (7)$$

where  $\beta$  is the meridional gradient of the Coriolis parameter, defined as  $\beta = \frac{2\Omega \cos \varphi}{R}$ ,  $\Omega$  is the Earth's angular velocity of rotation,  $\varphi$  is the latitude,  $R$  is the Earth's radius.

### 3.2.2 Density Neural Networks Module

155

The Density Neural Networks module incorporates multiple fully connected neural network (NN) layers, organized into NN submodules, and employs residual connections similar to ResNet (Razavi et al., 2024) to facilitate efficient information transfer between modules. This design aims to minimize the loss of information from the initial geostrophic current field during feature extraction and to accelerate model convergence and enhance stability. The AC-PIDNN architecture employs three NN blocks, each containing five hidden layers with 100 neurons per layer. Each NN block consists of a sequence of fully connected transformations:

160

$$h^{(l)} = \sigma(W^{(l)}h^{(l-1)} + b^{(l)}), \quad l = 1, \dots, L \quad (8)$$

where  $\mathbf{h}^{(0)} = \mathbf{X}$ ,  $W^{(l)}$  and  $b^{(l)}$  are the trainable weights and biases of the  $l$ -th layer,  $\sigma(\cdot)$  denotes the activation function, and  $L$  is the number of hidden layers in each block. To preserve the physical meaning of the geostrophic input and reduce the loss of dynamical information, a residual connection is applied:

165

$$h^{(l)} = f(h^{(l)}) + h^{(l-1)} \quad (9)$$

This residual mechanism allows the NN block to learn corrections to the geostrophic current field rather than re-learning it from scratch. From a physical perspective, the predicted current field  $\vec{V}_{total} = (u_{total}, v_{total})$  can be expressed as the sum of the geostrophic component and a correction term:

$$u_{total} = u_{geo} + \Delta u(\mathbf{X}; \theta) \quad (10)$$

170

$$v_{total} = v_{geo} + \Delta v(\mathbf{X}; \theta) \quad (11)$$

where  $\Delta u$  and  $\Delta v$  are the data-driven corrections parameterized by the network weights  $\theta$ . This formulation ensures that the essential dynamical information from the geostrophic approximation is retained while enabling the network to learn the ageostrophic corrections. Consequently, the residual structure accelerates convergence, improves gradient stability during backpropagation, and prevents the model from overfitting to spurious patterns in the training data.

### 175 3.2.3 Loss Function Module and Training Strategy

The loss function module consists of three customized loss functions. To address this and eliminate the need for manual hyperparameter tuning (e.g., grid search), we implemented an adaptive loss scaling mechanism during the Stage 2 hybrid training.

$$Loss_{data} = \frac{1}{N} \sum_{i=1}^N \|u_{Pseudo\ labels} - u_{total-i}\|^2 + \|v_{Pseudo\ labels} - v_{total-i}\|^2 \quad (12)$$

$$180 \quad Loss_{physics} = \frac{1}{N} \sum_{i=1}^N \|ADV_x + COR_x + PGF_x + WIND_x\|^2 + \|ADV_y + COR_y + PGF_y + WIND_y\|^2 \quad (13)$$

$$Loss_{hybird} = Loss_{data} + \lambda \times Loss_{physics} \quad (14)$$

Here,  $N$  denotes the number of training samples;  $\vec{V}_{Pseudo\ labels} = (u_{Pseudo\ labels}, v_{Pseudo\ labels})$  represents the pseudo-labels for model initialization, which are constructed using the sum of the corresponding geostrophic and Ekman components; and  $\vec{V}_{total-i} = (u_{total-i}, v_{total-i})$  are the model-predicted current components in the  $i - th$  epoch;  $Loss_{data}$  represents the mean squared error between the predicted and geostrophic currents, and is used for initial model fitting;  $Loss_{physics}$  measures the residuals of the predicted current field in the momentum equations, constraining the outputs to satisfy the two-dimensional momentum balance as closely as possible; It is worth noting that the  $Loss_{physics}$  is calculated for each predicted  $(u_{total-i}, v_{total-i})$ , which is a dynamic value.  $Loss_{hybird}$  is a weighted combination of the above two, enabling a smooth transition from data-driven to physics-driven learning by gradually increasing the weight of physical constraints during training.

190 The adaptive weight  $\lambda$  is calculated as  $\lambda = \text{sg}\left(\frac{L_{data}}{L_{physics}}\right)$ . Here,  $\text{sg}(\cdot)$  denotes the stop-gradient operation, ensuring that the weight acts only as a scaling factor and does not interfere with the backpropagation process. This mechanism maintains a 1:1 balance between data and momentum constraints throughout the training, ensuring robustness and reproducibility.

The adaptive optimization algorithm Adam (Kingma and Ba, 2014) is employed to minimize the loss function, and neural network parameters are initialized using the Xavier scheme (Glorot and Bengio, 2010). The hyperbolic tangent ( $\tanh$ ) is adopted as the activation function. Furthermore,  $L_2$  regularization and dropout are utilized to mitigate overfitting, with each serving distinct physical roles within the architecture. Once training converges and the loss function reaches a sufficiently small value, the predicted  $(u_{total}, v_{total})$  values approximate the solutions of the momentum equations (1) and (2). To ensure stability and accuracy, a three-stage training strategy is adopted:

200 Stage 1 (Data-Driven Initialization): Using only  $Loss_{data}$  (Learning rate = 0.001, Epochs = 1000). This allows the model to rapidly capture the macroscopic features of the geostrophic and Ekman flows. It establishes a physically reasonable starting point, preventing the non-uniqueness problem that occurs when starting from random initialization.

Stage 2 (Transitional Hybrid Optimization): Using  $Loss_{hybird}$  (Learning rate = 0.0001, Epochs = 1000). Enforcing strong physical constraints too abruptly can cause gradient instability. This hybrid stage acts as a buffer, gently introducing momentum balance constraints to guide the model away from purely data-fitted local minima.

205 Stage 3 (Pure Physics Refinement): Using only  $Loss_{physics}$  (Learning rate = 0.00001, Epochs = 10000). By removing the data constraint, we allow the network to minimize the momentum residuals strictly. Because the model is already anchored in a realistic state by Stages 1 and 2, minimizing  $Loss_{physics}$  does not lead to an arbitrary solution; instead, it accurately isolates and resolves the non-linear dynamical components that cannot be captured by data fitting alone.

210 Notably, to successfully isolate the ageostrophic components from the momentum equations, the combined geostrophic and Ekman flow components are utilized as pseudo-labels during Stages 1 and 2. Subsequently, Stage 3 relies entirely on physical constraints to execute the separation of the ageostrophic components. Detailed parameter configurations for the proposed AC-PIDNN model are provided in Supplementary Table S1.

### 3.3 Ageostrophic Current Component Decomposition

The ocean current field  $V_{total}$  predicted by PINN contains the component of geostrophy ( $\vec{V}_{geo}$ ), wind stress ( $\vec{V}_{ekman}$ ), and 215 nonlinearity ( $\vec{V}_{adv}$ ):

$$\vec{V}_{total} = \vec{V}_{geo} + \vec{V}_{adv} + \vec{V}_{ekman} \quad (15)$$

Where  $\vec{V}_{total}$  is the velocity vector predicted by AC-PIDNN,  $\vec{V}_{geo}$  is the geostrophic current from AVISO data and  $\vec{V}_{ekman}$  is surface Ekman current in GLOBCURRENT data. Thus, the ADV, COR and WIND effect in Equation (1-2) is represented as  $f\vec{k} \times \vec{V}_{adv}$ ,  $f\vec{k} \times \vec{V}_{total}$ , and  $f\vec{k} \times \vec{V}_{ekman}$ . To verify the balance of the predicted  $u_{total}$  and  $v_{total}$  from AC-PIDNN, the 220 following balance was examined:

$$Bal_x = COR_x + PGF_x + WIND_x + ADV_x \quad (16)$$

$$Bal_y = COR_y + PGF_y + WIND_y + ADV_y \quad (17)$$

$$Bal = \left( \left( \frac{Bal_x}{PGF_x} \right)^2 + \left( \frac{Bal_y}{PGF_y} \right)^2 \right)^{\frac{1}{2}} \quad (18)$$

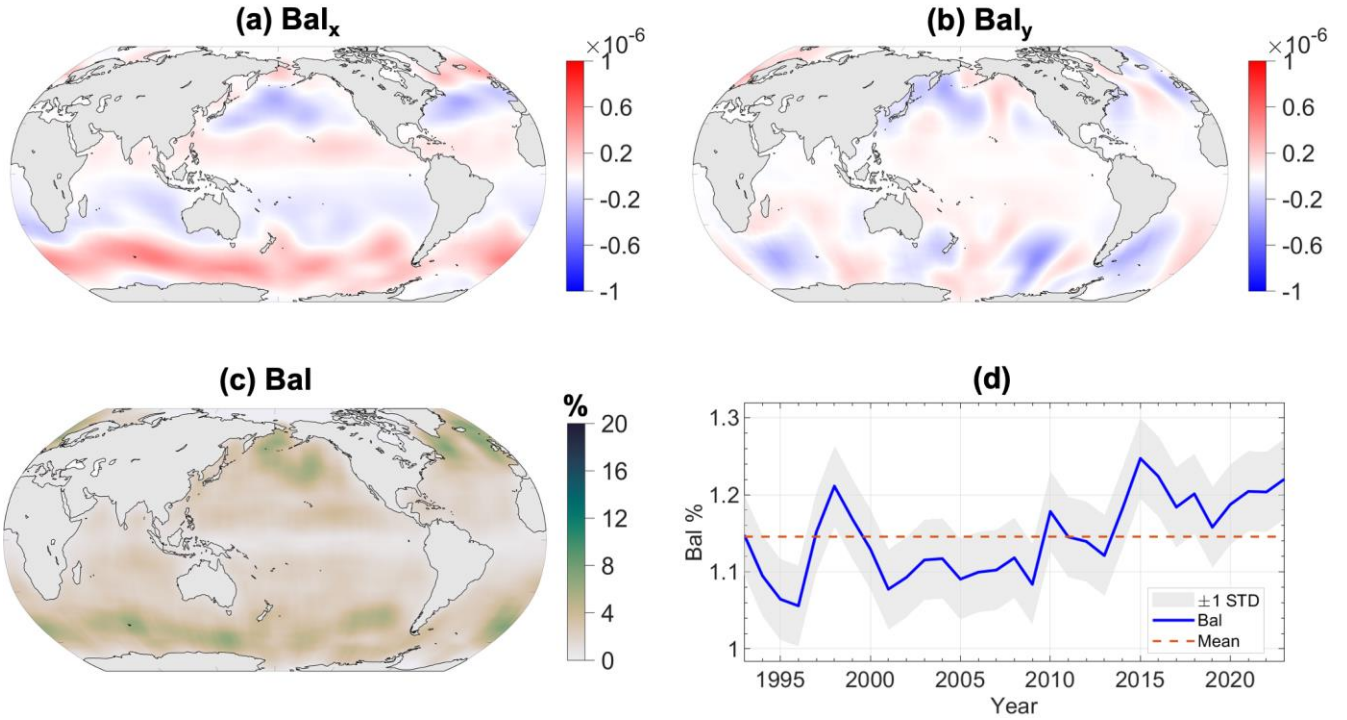
225 Which quantifies the normalized residual of the balance in Equation (1-2). It is worth noting that, the  $Bal$  in this study is not intended as an independent external validation metric, but rather as a core self-consistency standard ensuring internal physical consistency. The minimization of this residual strictly validates that the extracted non-linear advection terms satisfy the physical equation constraints.

## 4 Results and Discussions

### 4.1 Validation of reconstructed currents and dynamical consistency

#### 230 4.1.1 Dynamical consistency

This study employed the AC-PIDNN to reconstruct the daily global sea surface circulation from 1993 to 2023, incorporating geostrophic, nonlinear advective and Ekman-corrected components. The 30-year mean dynamical consistency of the AC-PIDNN-reconstructed surface circulation was evaluated by examining the zonal ( $Bal_x$ ) and meridional ( $Bal_y$ ) balances, together with the ratio of the residual magnitude to the pressure gradient force (Bal) (Fig.2 (a-c)). The residuals are generally small over most of the global ocean, with typical magnitudes on the order of  $10^{-6} \text{ m s}^{-2}$ . Relatively larger residuals are mainly confined to regions characterized by strong currents and energetic mesoscale variability, including western boundary current extensions and parts of the Southern Ocean. Despite these localized enhancements, the overall balance index Bal remains below 5%, indicating that the reconstructed surface circulation satisfies the surface momentum balance to a high degree across the global ocean. The Fig.2 (d) illustrates the temporal evolution of the globally averaged Bal from 1993 to 2023. The global mean value remains stable over the entire period, with the global mean  $\pm$  standard deviation range varies between 1%~1.5%. No systematic drift or long-term trend is evident, demonstrating that the correction maintains a consistent level of dynamical balance throughout the multi-decadal record. Figures S1 and S2 in the Supplementary Material show the momentum balance (Bal) evolution from Stage 1 to Stage 3 of the AC-PIDNN model's training, as well as comparisons with in-situ measurements and the Ocean Surface Current Analyses Real-time (OSCAR) dataset, validating the strong accuracy of the AC-PIDNN corrections. Together, these results confirm that the AC-PIDNN correction yields a surface circulation that is dynamically self-consistent and temporally robust.



**Figure 2: Balance analysis of the corrected 30-year means flow field within the momentum equations (4) and (5). Panel (a) 30-year mean residual of the zonal momentum equation,  $Bal_x$  ( $m s^{-2}$ ), from 1993 to 2023; Panel (b) 30-year mean residual of the meridional momentum equation,  $Bal_y$ , from 1993 to 2023; Panel (c) 30-year mean ratio of the magnitude of the momentum equation residuals to the pressure gradient force,  $Bal$ , from 1993 to 2023. Panel (d) Time evolution of the globally averaged  $Bal$  from 1993 to 2023, with the shaded area representing the  $\pm 1$  standard deviation range.**

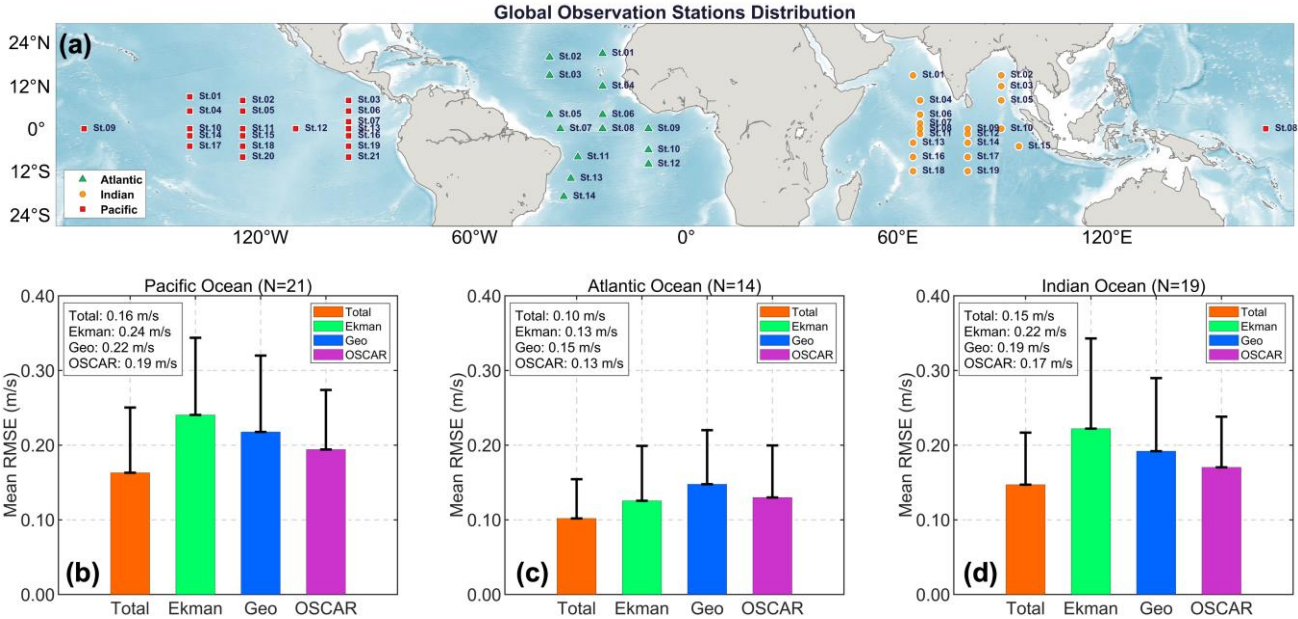
#### 4.1.2 Comparison of AC-PIDNN Corrected Flow Fields and OSCAR Products Against In-Situ Data

To comprehensively validate the reliability and applicability of the AC-PIDNN dataset across different dynamical regimes globally, a systematic evaluation was conducted using widespread in-situ observations, alongside a multi-region comparative analysis with the existing OSCAR surface current product (1993-2022) downloaded from <https://www.esr.org/data-products/oscar/>. The in-situ data used for validation are derived from the tropical mooring buoy arrays, comprising the TAO array in the Pacific Ocean, the PIRATA array in the Atlantic Ocean, and the RAMA array in the Indian Ocean. As shown in Figure 3(a), a total of 54 observation stations widely distributed across the Pacific Ocean (21 stations), Atlantic Ocean (14 stations), and Indian Oceans (19 stations) (downloaded from <https://www.pmel.noaa.gov/tao/drupal/disdsl/>).

The Root Mean Square Error (RMSE) was utilized to evaluate the AC-PIDNN-corrected total currents, OSCAR currents, geostrophic currents (Geo), and Ekman-corrected currents (Figure 3(b)–(d)). Statistical results demonstrate that across all three major ocean basins, the surface circulations corrected by AC-PIDNN are significantly closer to the in-situ observations compared to the traditional geostrophic and Ekman-corrected components. Specifically, in the Pacific Ocean, the RMSE of



the AC-PIDNN-corrected current is 0.16 m/s, which outperforms the Ekman-corrected (0.24 m/s), geostrophic (0.22 m/s), and standard OSCAR product (0.19 m/s) currents, achieving an error reduction of approximately 15.8% relative to OSCAR. In the Indian Ocean, the AC-PIDNN RMSE is 0.10 m/s, outperforming both the Ekman-corrected (0.13 m/s) and geostrophic (0.15 m/s) currents, with a 23.1% error reduction compared to OSCAR (0.13 m/s). In the Atlantic Ocean, the AC-PIDNN RMSE is 0.15 m/s, which is lower than the Ekman-corrected (0.22 m/s) and geostrophic (0.17 m/s) currents, representing an 11.8% error reduction compared to OSCAR (0.17 m/s). Furthermore, the supplementary material provides time-series comparisons of the in-situ measured currents against the AC-PIDNN-corrected flow fields, OSCAR products, as well as the geostrophic and Ekman-corrected components at 28 stations. In summary, the basin-wide validation using these 54 independent stations conclusively demonstrates that the AC-PIDNN dataset delivers consistent and robust accuracy across all three major global oceans.



**Figure 3: (a) Spatial distribution of TAO/PIRATA/RAMA mooring sites across the Pacific, Atlantic, and Indian Oceans. In-situ observation stations in each ocean basin are individually numbered to facilitate subsequent comparative experiments. (b)–(c) Bar charts illustrating the averaged root-mean-square error (RMSE) between TAO/PIRATA/RAMA mooring observations, OSCAR products and the AC-PIDNN-corrected current fields, geostrophic currents, and Ekman-corrected currents for the Pacific, Atlantic, and Indian Oceans, respectively.**

To further evaluate the spatial distribution characteristics of the errors, a latitudinal error analysis of the AC-PIDNN and OSCAR surface current products was conducted using the in-situ observations. As illustrated in Fig.4, within the latitudinal range of approximately  $\pm 25^\circ$  (corresponding to the spatial coverage of the deployed observation arrays), the AC-PIDNN-corrected dataset consistently outperforms the OSCAR product in matching the in-situ measurements. The analysis reveals

that larger errors are predominantly concentrated in the equatorial band (within a latitude range of approximately  $\pm 5^\circ$ ). Conversely, accuracy improves significantly in mid-to-low latitudes and regions further from the equator, aligning much more closely with field observations. Specifically, the RMSE decreases gradually from 0.25 m/s near the equator to 0.05 m/s in the higher-latitude observation areas. Overall, within the  $\pm 25^\circ$  latitudinal band, the AC-PIDNN product consistently demonstrates better performance than the OSCAR product, indicating that the improved reliability of the dataset is not an isolated artifact but a geographically consistent feature across a broad spatial extent. Additionally, it should be noted that due to the limited spatial coverage of current continuous in-situ observation arrays, there is presently a lack of robust comparison in high-latitude regions.

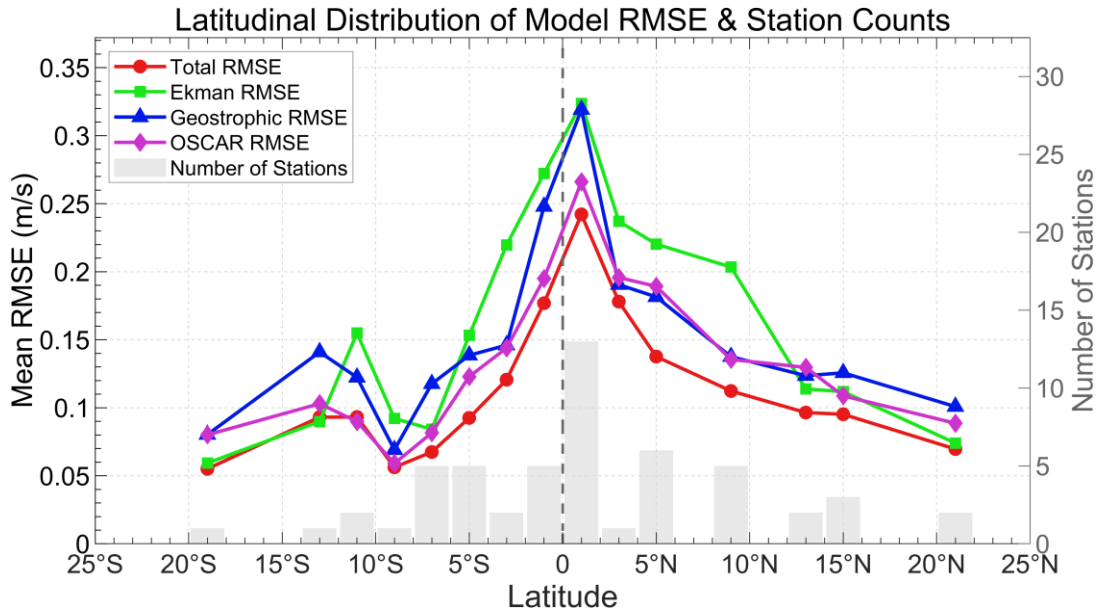
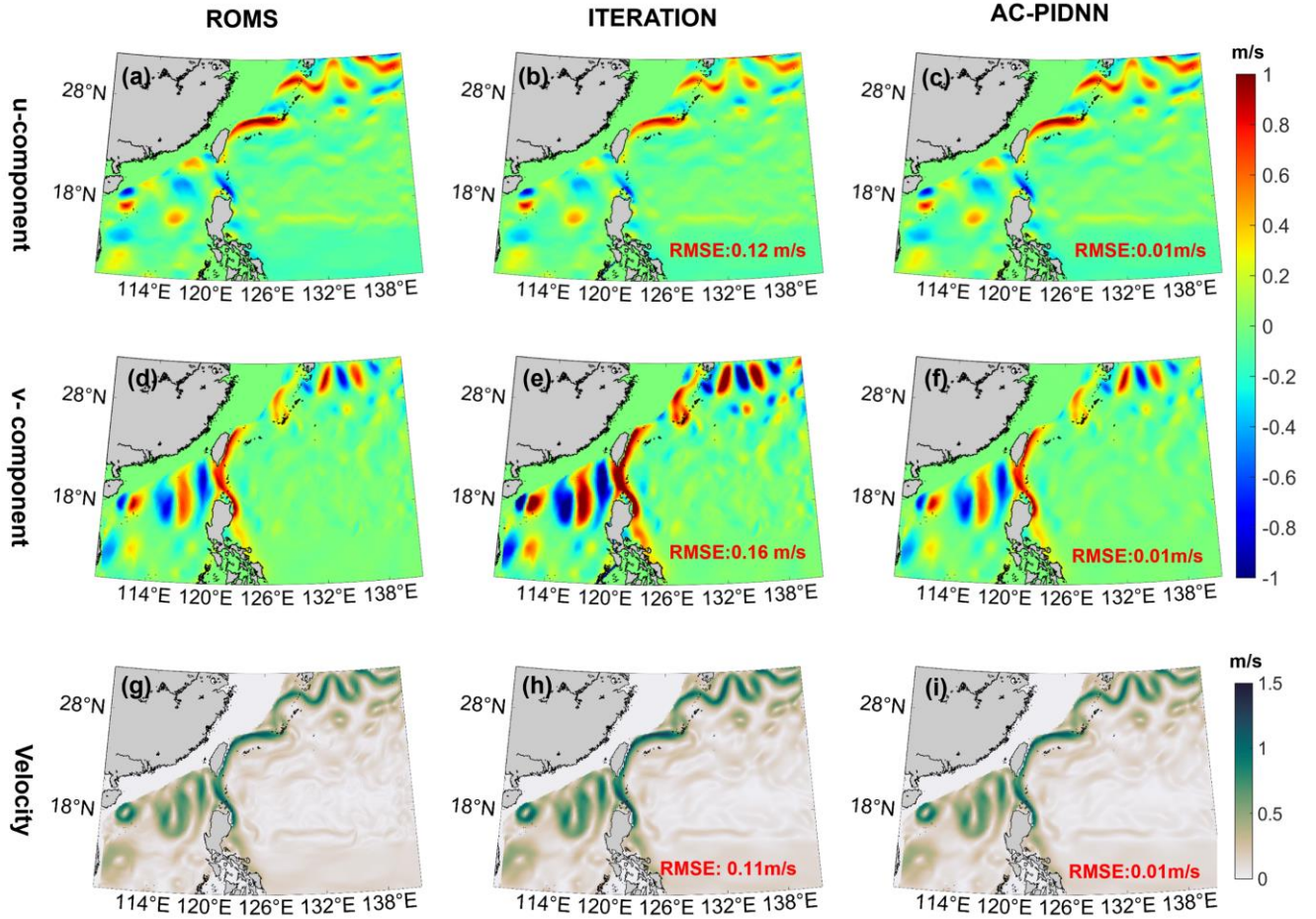


Figure 4: presents the latitudinal error curves of the in-situ measurements against both the corresponding AC-PIDNN-corrected currents and the OSCAR products across the 54 observation stations.

#### 4.1.3 Comparison of AC-PIDNN Corrected Flow Fields Against ROMS data

To further evaluate the reconstruction accuracy, we conducted an additional validation using a high-resolution ROMS simulation as an independent reference. In this experiment, the ROMS-simulated sea surface height was used as the sole input to reconstruct the surface velocity field, ensuring a fair comparison between the baseline iterative method and the AC-PIDNN approach. As the ROMS simulation used herein neglects the Ekman component, this component is not taken into account in the present study. Figure 5 compares the reconstructed surface velocity fields with the ROMS reference in the northwestern Pacific Ocean. The zonal ( $u$ ) and meridional ( $v$ ) velocity components, as well as the velocity magnitude, are shown for the ROMS simulation, the Iteration method, and the AC-PIDNN reconstruction. The Iteration method reproduces the general

structure of the velocity fields but exhibits substantial amplitude mismatches, particularly in regions of strong currents and mesoscale activity, with RMSE values of  $0.12 \text{ m s}^{-1}$  for  $u$  and  $0.16 \text{ m s}^{-1}$  for  $v$ , and  $0.11 \text{ m s}^{-1}$  for the magnitude of the velocity over the mapped domain. In contrast, the AC-PIDNN reconstruction reproduces the dominant spatial structures of the surface circulation more faithfully. The pathway and intensity of the western boundary current, as well as the surrounding mesoscale features, are captured with improved spatial coherence. Correspondingly, the RMSE is reduced to approximately  $0.01 \text{ m s}^{-1}$  for both velocity components and for the velocity magnitude, representing nearly an order-of-magnitude improvement over the iterative approach. Together with the momentum-balance diagnostics, the ROMS-based validation confirms that the corrected surface circulation is both dynamically consistent and quantitatively accurate under controlled conditions.



**Figure 5: Sea Surface Flow Field after Correction of Sea Surface Height Anomalies in the Northwestern Pacific using Iterative Method and AC-PIDNN.** The first column shows the u-component, v-component, and flow field of the simulated data; the second column show the results corrected by the iterative method and AC-PIDNN method.

## 4.2 Characteristics of the corrected surface circulation

320 In this section, we decompose the daily surface circulation (1993–2023) into geostrophic ( $\vec{V}_{geo}$ ), Ekman-corrected ( $\vec{V}_{Ekman}$ ), nonlinear advective ( $\vec{V}_{adv}$ ) components. The detailed results for the Kuroshio region and the equatorial Pacific region are shown in Fig.S4 and S5, respectively. For each component, we analysed the zonal and meridional velocity components ( $u$  and  $v$ ) together with the flow speed (Fig. 6).

325 The nonlinear advective component (Fig.6 (a–c)) exhibits a relatively smaller magnitude. Both its zonal and meridional contributions are generally on the order of  $10^{-2} \text{ m s}^{-1}$ , with elevated values confined to dynamically active regions. These localized enhancements are most evident along the equatorial band, within western boundary current systems and their extensions, and over parts of the Southern Ocean (Qiu et al., 2014). Over the majority of the open ocean, the advective component remains weak, indicating that its contribution to the corrected surface circulation is primarily expressed through regional and localized modulation rather than basin-scale influence. The Ekman-corrected component (Fig.6 (d–f)) displays a  
330 clear large-scale organization. Its zonal and meridional velocities show pronounced latitudinal banding over wide areas of the tropics and subtropics, consistent with the distribution of large-scale wind forcing. Typical Ekman-corrected velocities reach  $O(10^{-1}) \text{ m s}^{-1}$  in low- and mid-latitude regions, with reduced magnitudes toward higher latitudes.

The large-scale structure and intensity of the surface circulation are primarily controlled by the geostrophic flow (Fig.6 (g–i)). Both zonal and meridional geostrophic velocities exhibit strong signals associated with major current systems, including  
335 western boundary currents and their extensions and the Antarctic Circumpolar Current. The geostrophic speed commonly reaches  $O(10^{-1} - 10^0) \text{ m s}^{-1}$ . The weaker values are found in the interiors of subtropical gyres. As a result of the combined contributions, the total surface velocity field (Fig.6 (j–l)) closely resembles the geostrophic circulation in its overall structure, while exhibiting systematic modifications in both magnitude and spatial details. In particular, westward zonal flow near the equator and Southern Ocean, and meridional transport in adjacent regions are strengthened relative to a purely geostrophic  
340 approximation, highlighting the cumulative influence of Ekman-corrected and nonlinear advective processes in shaping the corrected surface circulation. Neglecting these ageostrophic components would therefore bias the spatial distribution and magnitude of reconstructed surface currents.

Because the total velocity is defined as the linear sum of the three components, this decomposition provides a natural basis for examining their respective contributions to commonly used kinematic and energetic diagnostics, as discussed in the  
345 following section.



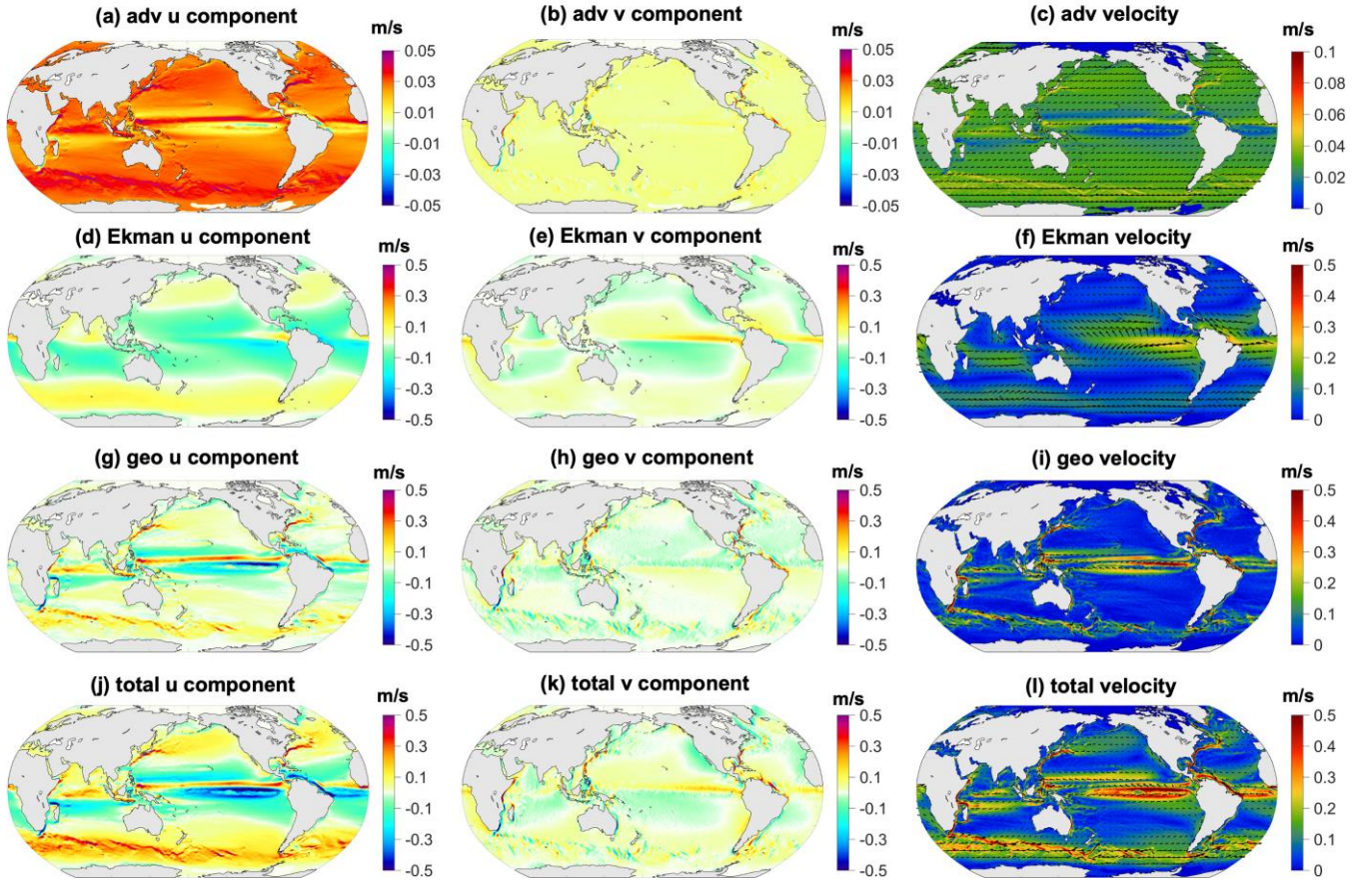


Figure 6: Results of Sea Surface Circulation Correction from 1993 to 2023, using the AC-PIDNN model, incorporating geostrophic, nonlinear advective and Ekman-corrected components. Panels (a)–(c) show the u-component, v-component, and flow field of the nonlinear advective component; Panels (d)–(f) show the u-component, v-component, and flow field of the Ekman-corrected component; Panels (g)–(i) show the u-component, v-component, and flow field of the geostrophic component; Panels (j)–(l) show the u-component, v-component, and flow field of the total circulation.

### 4.3 Impacts on kinematic and energetic diagnostics

This section examines the impacts of the velocity correction on commonly used kinematic and energetic diagnostics, including surface relative vorticity (VOR), divergence (DIV), and eddy kinetic energy (EKE). These variables are widely applied in studies of upper-ocean dynamics (Wunsch, 2025) and therefore provide an important perspective on the implications of the corrected surface circulation.

$$VOR = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad (19)$$

$$DIV = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \quad (20)$$

$$EKE = \frac{1}{2}(u'^2 + v'^2) \quad (21)$$

### 4.3.1 Vorticity

Sea surface relative vorticity is a key dynamical parameter that describes the rotational characteristics of the upper ocean flow, directly linked to flow shear, vortex structures, and angular momentum distribution (Zhang et al., 2024; Kwak and Richmond, 2014). The geostrophic component (Fig.7 (c)) dominates the large-scale structure of the relative vorticity field. Regions of strong positive and negative vorticity are closely aligned with major current systems, including western boundary currents, their downstream extensions, and the Antarctic Circumpolar Current. This distribution outlines the primary rotational framework of the surface circulation and reflects the strong shear associated with geostrophic jets and frontal zones. In addition to the geostrophic component, the wind-driven Ekman flow contributes significantly to the sea surface relative vorticity. Here, the Ekman-corrected component refers specifically to the vorticity generated by the velocity field that balances the wind stress forcing in the horizontal momentum equations (i.e., the Ekman flow), as distinct from the vertical Ekman pumping velocity associated with wind stress curl (Chen et al., 2020). While generally weaker in magnitude than geostrophic vorticity, this component exhibits distinct regional signatures. Enhanced Ekman vorticity (Fig.7 (b)) is particularly evident in tropical and subtropical basins where winds are strong and relatively steady, such as the trade wind belts and prevailing westerlies, where lateral shear in the Ekman-corrected velocity can generate significant vorticity signals (Chen et al., 2020). For instance, in equatorial regions, zonal wind stress can excite pronounced Ekman vorticity anomalies due to the variation of the Coriolis parameter. In contrast, the nonlinear vorticity (Fig.7 (a)) term arising from advection by the flow itself is typically about two orders of magnitude smaller in strength than the geostrophic and -corrected components. However, in regions of intense currents and strong shear, such as the Kuroshio Extension and Gulf Stream separation zones, nonlinear processes become crucial in modulating vorticity variability, energy cascade, and submesoscale vortex activities, and thus cannot be neglected (Chen et al., 2020). We present the 30-year average vorticity for the western boundary Kuroshio region and the wind-forced equatorial Pacific current region in Supplementary Fig.S6 (a-d) and Fig.S7 (a-d), respectively. This indicates that while the foundational structure of the large-scale vorticity field is set by geostrophic balance, with wind forcing and nonlinear dynamics providing an additional contribution to vorticity magnitude particularly in dynamically active regions.

### 4.3.2 Divergence

Sea surface horizontal divergence is a key physical quantity characterizing the divergent/convergent nature of the upper-ocean flow, with important implications for mass transport (Dubois et al., 2016). Unlike the vorticity field, the dynamical components of the divergence field exhibit distinct and contrasting contributions. From a dynamical decomposition perspective, the horizontal divergence of the geostrophic component (Fig.7 (g)) is negligible at the sea surface under the continuum layer

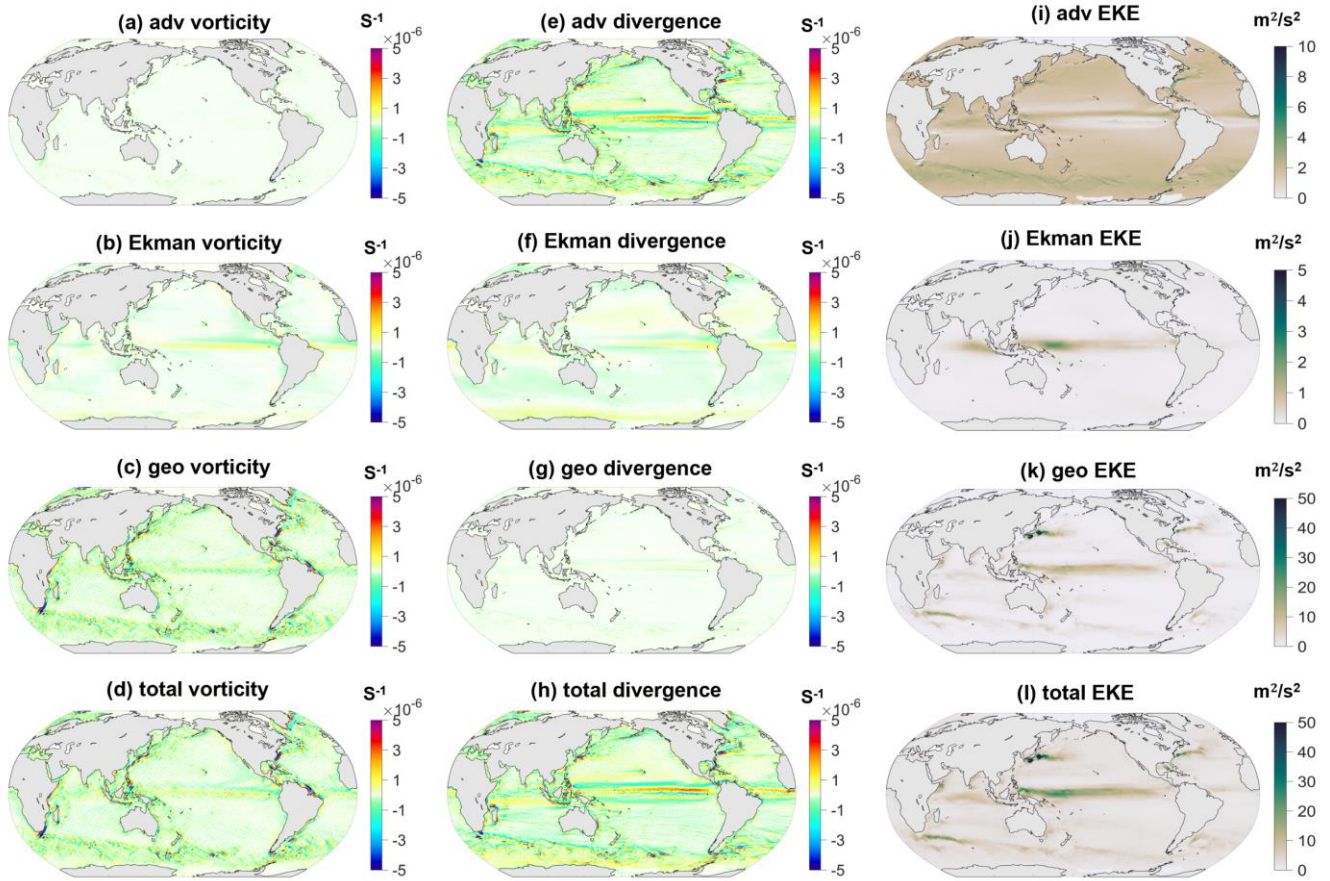
approximation, dictated by its non-divergent nature. Thus, the geostrophic flow contributes insignificantly to the large-scale  
390 sea surface horizontal divergence. In contrast, the nonlinear advective component (Fig.7 (e)) dominates the large-scale spatial  
pattern of the divergence field. Its magnitude is substantially larger than other components, forming prominent bands of  
divergence/convergence in regions of strong currents (e.g., western boundary current extensions) and vigorous eddy activity.  
It is noteworthy that although the local amplitude of the advective divergence is large, its net contribution to basin-scale or  
global integrated divergence is modest due to the cancellation between extensive positive and negative regions. The Ekman-  
395 corrected component of divergence (Fig.7 (f)), however, originates directly from Ekman pumping/suction driven by wind  
stress curl. Its local intensity is typically weaker than the advective component. Due to its more spatially coherent sign (e.g.,  
divergence over regions of positive wind stress curl and convergence over negative curl regions), the Ekman-corrected  
component contributes significantly to the domain-averaged or net divergent effect at regional or global scales, and its  
magnitude is non-negligible. For instance, in the major wind belts (trade winds, westerlies), Ekman divergence forms a key  
400 driver for large-scale vertical circulation cells, such as the subtropical gyres (Roquet et al., 2025). [To further illustrate the  
impacts of nonlinear processes and wind forcing on sea surface divergence, we present the 30-year average vorticity for the  
western boundary Kuroshio region and the wind-forced equatorial Pacific current region in Supplementary Fig.S6 \(e-h\) and  
Fig.S7 \(e-h\), respectively.](#)

#### 4.3.3 Eddy Kinetic Energy

405 Eddy Kinetic Energy is an integrated metric for the intensity of mesoscale variability, representing the perturbation energy  
of the flow velocity field relative to its temporal mean. It serves as a key parameter for assessing the energetic consistency and  
dynamical plausibility of the corrected velocity field (Huo et al., 2024). As shown in Fig.7 (i-l) the global EKE distribution  
derived from the corrected circulation field, which incorporates geostrophic, Ekman-corrected, and nonlinear advective  
components exhibits the well-documented pattern of high-energy regimes: primarily concentrated in western boundary current  
410 extensions (e.g., the Kuroshio Extension, Gulf Stream Extension), the core regions of the Antarctic Circumpolar Current, and  
equatorial current systems, all areas of high dynamical activity (Su et al., 2023). To further illustrate this point, Supplementary  
Fig.S6 (i-l) and Fig.S7 (i-l) present the 30-year mean EKE characteristics in the equatorial Pacific region and the Kuroshio  
region, respectively. These energetic hotspots are closely linked to strong shear, instability processes, and active mesoscale  
eddy generation. Compared to EKE estimates based solely on the geostrophic approximation, the inclusion of the Ekman and  
415 nonlinear advective components does not introduce spurious amplification of the total kinetic energy. Instead, it redistributes  
the energy spatially in a physically consistent manner. The correction primarily modulates the structure of the EKE field rather  
than significantly altering its global intensity magnitude. Specifically, in regions where wind forcing is pronounced, such as  
the equatorial and subtropical zones, incorporating the Ekman-corrected component provides a more realistic representation  
of fluctuating energy directly generated by wind variability. Meanwhile, in the vicinity of strong current edges and frontal  
420 zones, nonlinear advective processes dominate in shaping the fine-scale structure of EKE, such as energy cascade and eddy-  
mean flow interactions (Adeagbo et al., 2022). In summary, the multi-component velocity correction scheme redistributes the



spatial pattern of EKE in a manner consistent with physical mechanisms while preserving the overall energetic framework of the surface circulation. This enhances the capability of the EKE field to characterize mesoscale processes, energy transfer pathways, and mixing efficiency, yielding a representation that is closer to the true dynamical context of the ocean.



**Figure 7: 30-year average global sea surface dynamic parameter characteristics. Panels (a)–(d) showing the vorticity of the advective, Ekman-corrected, geostrophic, and total components, respectively; Panels (e)–(h) showing the divergence of the advective, Ekman-corrected, geostrophic, and total components, respectively; Panels (i)–(l) showing the eddy kinetic energy (EKE) of the advective, Ekman-corrected, geostrophic, and total components, respectively.**

#### 4.3.4 Temporal Evolution of Dynamical Diagnostics

To analyze the long-term variations in the three dynamic parameters mentioned above, Fig.8 presents the 30-year averaged time series for the global ocean, the Kuroshio region (110°E–175°E, 12°N–50°N), and the Equatorial Current region (60°W–110°E, 20°S–20°N). On a global scale (Fig.8 (a-c)), the interannual variability of total divergence ( $O(10^{-8})$  s<sup>-1</sup>) is



435 primarily dominated by the Ekman-corrected component ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ), with the nonlinear component ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ) also contributing significantly, and their variation trends are largely consistent. The magnitude of variability in nonlinear divergence is smaller than that of the Ekman-corrected component but significantly larger than the negligible geostrophic divergence ( $\mathcal{O}(10^{-10}) \text{ s}^{-1}$ ). The total relative vorticity field ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ) is mainly determined by the combined effects of the geostrophic component ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ) and the Ekman-corrected component ( $\mathcal{O}(10^{-10}) \text{ s}^{-1}$ ). In terms of trends, the Ekman-corrected vorticity  
 440 exhibits a weakening trend, whereas both geostrophic and total vorticities show a long-term enhancement of negative vorticity. No significant interannual signal is detected in nonlinear vorticity. Regarding eddy kinetic energy (EKE), the trend in total EKE aligns with that of geostrophic EKE but is notably modulated by nonlinear processes. Moreover, the magnitudes of the nonlinear and geostrophic components are comparable, with the nonlinear EKE being higher in most years, yet both remain within the same order of magnitude ( $\mathcal{O}(10^0\text{--}10^1) \text{ m}^2 \text{ s}^{-2}$ ).

445 On a regional scale, two representative areas, the Kuroshio region and the Equatorial Current region are selected for analysis (Fig.8 (d-i)). In the Kuroshio region, the variation in total divergence follows the trend of the nonlinear component, although the contribution of the Ekman-corrected component is non-negligible, particularly in equatorial areas. The magnitude of divergence in the Kuroshio region ( $\mathcal{O}(10^{-8}) \text{ s}^{-1}$ ) is larger than that in the Equatorial Current region ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ). Regarding vorticity, the Kuroshio region is dominated by negative vorticity, with the absolute value of the Ekman-corrected component  
 450 being significantly larger than that of the geostrophic component. In contrast, the Equatorial Current region exhibits positive Ekman-corrected vorticity and negative geostrophic vorticity, with total vorticity primarily contributed by the Ekman-corrected component. Additionally, the magnitudes of geostrophic and total vorticities in the Kuroshio region ( $\mathcal{O}(10^{-8}) \text{ s}^{-1}$ ) exceed those in the Equatorial Current region ( $\mathcal{O}(10^{-9}) \text{ s}^{-1}$ ). For EKE, variations in both regions are mainly governed by the geostrophic component. The trend in total EKE in the Equatorial Current region closely resembles that of its geostrophic component,  
 455 whereas the Kuroshio region has shown an increasing trend in EKE since approximately 2017. In terms of magnitude, total EKE, geostrophic EKE, and nonlinear EKE are all on the order of  $\mathcal{O}(10^0\text{--}10^1) \text{ m}^2 \text{ s}^{-2}$ . The Ekman-corrected EKE in the Equatorial Current region ( $\mathcal{O}(10^{-1}) \text{ m}^2 \text{ s}^{-2}$ ) is larger than that in the Kuroshio region ( $\mathcal{O}(10^{-2}) \text{ m}^2 \text{ s}^{-2}$ ).

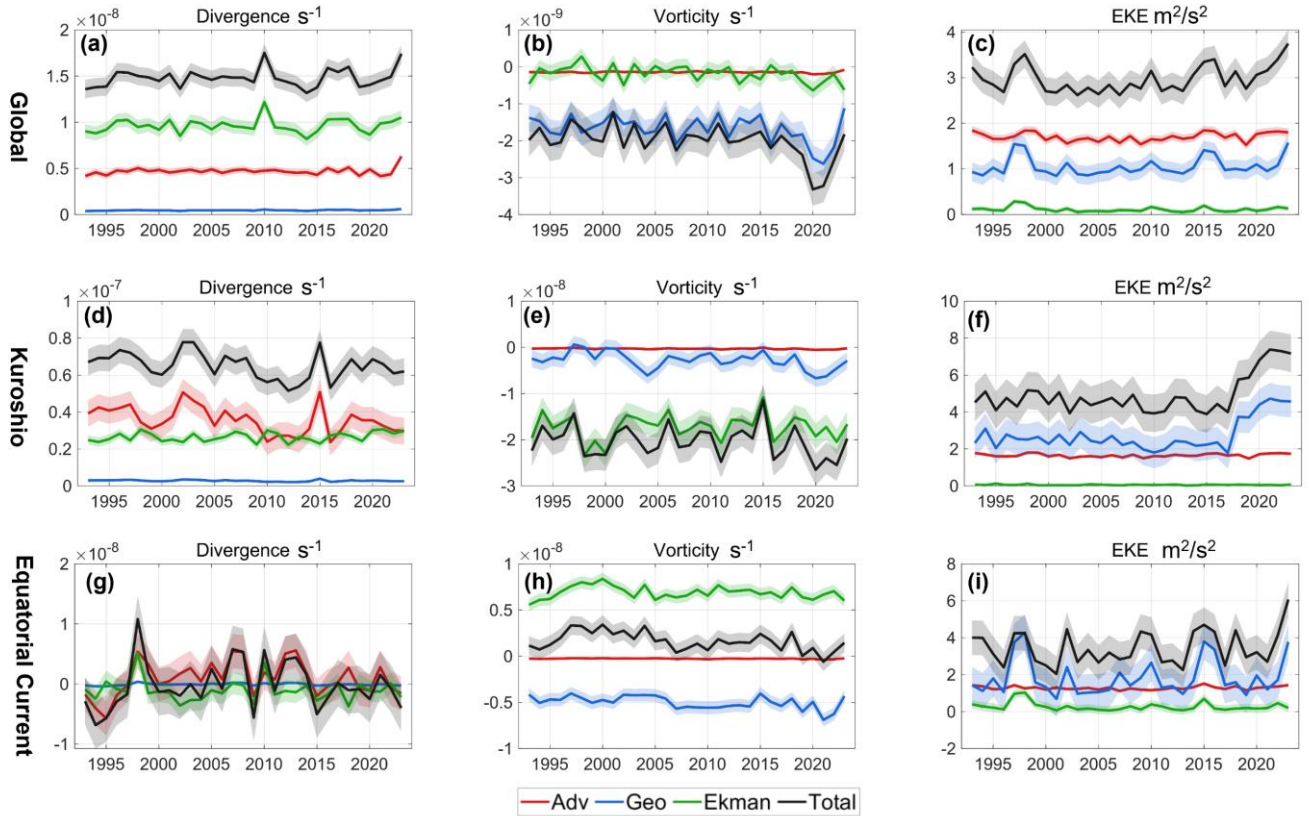


Figure 8: The annual mean variations in sea surface divergence, vorticity, and EKE from 1993 to 2023 for the global ocean, the Kuroshio region, and the equatorial current region. Panels (a)–(c) illustrate the interannual changes in globally averaged sea surface divergence, vorticity, and EKE; Panels (d)–(f) show the corresponding variations in the Kuroshio region ( $110^{\circ}$ – $175^{\circ}$ E,  $12^{\circ}$ – $50^{\circ}$ N); and panels (g)–(i) display the changes in the equatorial current region ( $60^{\circ}$ W– $110^{\circ}$ E,  $20^{\circ}$ S– $20^{\circ}$ N). The shaded area around each curve represents the  $\pm 1$  standard deviation range.

## 5 Code and data availability

The code for AC-PIDNN is available at <https://github.com/cgxcgxcgxcg/AC-PIDNN>. The global sea surface circulation dataset corrected for the period 1993–2023 in this study can be accessed via <https://zenodo.org/records/19966716> (Cui and Cai, 2026). This dataset has a spatial resolution of  $0.25^{\circ}$ , and each daily flow field file contains the variable names and their descriptions as listed in Table 1. Furthermore, the table outlines the Dataset ID corresponding to the satellite data product (global gridded geostrophic currents and surface Ekman currents) obtained from the Copernicus Marine Service at: <https://data.marine.copernicus.eu/product>.

**Table 1 Dataset Variable Information**

| Values       | size     | definition                                                                                                                                                                       | Dataset ID                                     |
|--------------|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------|
| sla          | 720×1440 | Sea Level Anomaly, sourced from the AVISO dataset.                                                                                                                               | cmems_obs-                                     |
| ugeo         | 720×1440 | Zonal (u) component of the geostrophic current, sourced from the AVISO dataset.                                                                                                  | sl_glo_phy-ssh_my_allsat-l4-duacs-0.125deg_PID |
| vgeo         | 720×1440 | Meridional (v) component of the geostrophic current, sourced from the AVISO dataset.                                                                                             |                                                |
| uekman       | 720×1440 | Zonal (u) component of the surface (0 m) Ekman current, sourced from the global Copernicus Ocean (GLOBCURRENT) surface and 15-meter Ekman and geostrophic current datasets.      | cmems_obs-mob_glo_phy-cur_my_0.25deg_PID-      |
| vekman       | 720×1440 | Meridional (v) component of the surface (0 m) Ekman current, sourced from the global Copernicus Ocean (GLOBCURRENT) surface and 15-meter Ekman and geostrophic current datasets. | m                                              |
| uadv         | 720×1440 | Zonal (u) component of the nonlinear advective term.                                                                                                                             | -                                              |
| vadv         | 720×1440 | Meridional (v) component of the nonlinear advective term.                                                                                                                        | -                                              |
| utotal       | 720×1440 | Zonal (u) component of the total flow field corrected by the AC-PIDNN model.                                                                                                     | -                                              |
| vtotal       | 720×1440 | Meridional (v) component of the total flow field corrected by the AC-PIDNN model.                                                                                                | -                                              |
| resu<br>Resu | 720×1440 | Residual of the zonal (u) momentum (Navier-Stokes) equation.                                                                                                                     | -                                              |
| resv<br>Resv | 720×1440 | Residual of the meridional (v) momentum (Navier-Stokes) equation.                                                                                                                | -                                              |
| lon          | 1 × 1440 | Longitude coordinates, ranging from -179.75°E to 180°E with a 0.25° resolution.                                                                                                  | -                                              |
| lat          | 720×1    | Latitude coordinates, ranging from -89.75°N to 90°N with a 0.25° resolution.                                                                                                     | -                                              |

## 6 Conclusions

475 A key contribution of this study is the development of a physics-informed deep learning framework to correct sea surface currents and quantify their dynamical components. By explicitly incorporating physical constraints, the framework substantially improves the dynamical consistency of reconstructed surface circulation and yields a global daily dataset for 1993-2023. Our results show that, although ageostrophic components are generally smaller than the geostrophic flow, they

make important contributions to the total surface circulation, especially in energetic regions such as western boundary currents and the Kuroshio Extension. On a global scale, the mean velocity of the total flow field is on the order of  $O(10^{-2}-10^{-1})$  m s<sup>-1</sup>, with the Ekman-corrected, geostrophic, and nonlinear components each exhibiting magnitudes of  $O(10^{-2})$  m s<sup>-1</sup>. In particular, nonlinear contributions have limited influence on relative vorticity, as the dominant rotational structure is captured by the geostrophic circulation. In contrast, the nonlinear corrections strongly affect surface divergence and eddy kinetic energy, revealing aspects of ocean surface dynamics that are not captured by geostrophic circulation alone. These findings show that resolving ageostrophic dynamics is essential for a more complete understanding of global surface circulation and its variability.

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# Supplementary Materials

## Daily Global Sea Surface Ageostrophic Current Dataset from 1993–2023 via Physics-Informed Deep Learning

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Table S1. Hyperparameters and training configurations of the AC-PIDNN model.

| Category                                                            | Hyperparameter / Setting               | Value / Description     |
|---------------------------------------------------------------------|----------------------------------------|-------------------------|
| Network Configuration                                               | Optimizer                              | Adam                    |
|                                                                     | Weight Initialization                  | Xavier                  |
|                                                                     | Activation Function                    | Tanh                    |
| Regularization (to prevent overfitting)                             | Dropout Rate                           | 0.3                     |
|                                                                     | L2 Regularization Factor ( $\lambda$ ) | 0.01                    |
| Stage 1: Initialization<br>(Quickly fit geostrophic current)        | Loss Function                          | Loss <sub>data</sub>    |
|                                                                     | Learning Rate                          | 0.001                   |
|                                                                     | Epochs                                 | 1000                    |
| Stage 2: Transition<br>(Gradually introduce physical constraints)   | Loss Function                          | Loss <sub>hybrid</sub>  |
|                                                                     | Hybrid Loss Proportion                 | 1:1                     |
|                                                                     | Learning Rate                          | 0.0001                  |
|                                                                     | Epochs                                 | 1000                    |
| Stage 3: Physics-Informed<br>(Minimize momentum equation residuals) | Loss Function                          | Loss <sub>physics</sub> |
|                                                                     | Learning Rate                          | 0.00001                 |
|                                                                     | Epochs                                 | 10000                   |

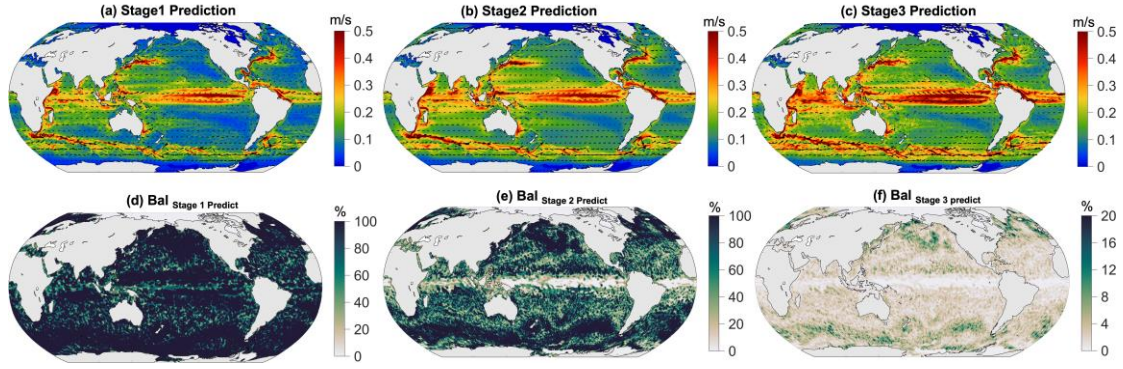
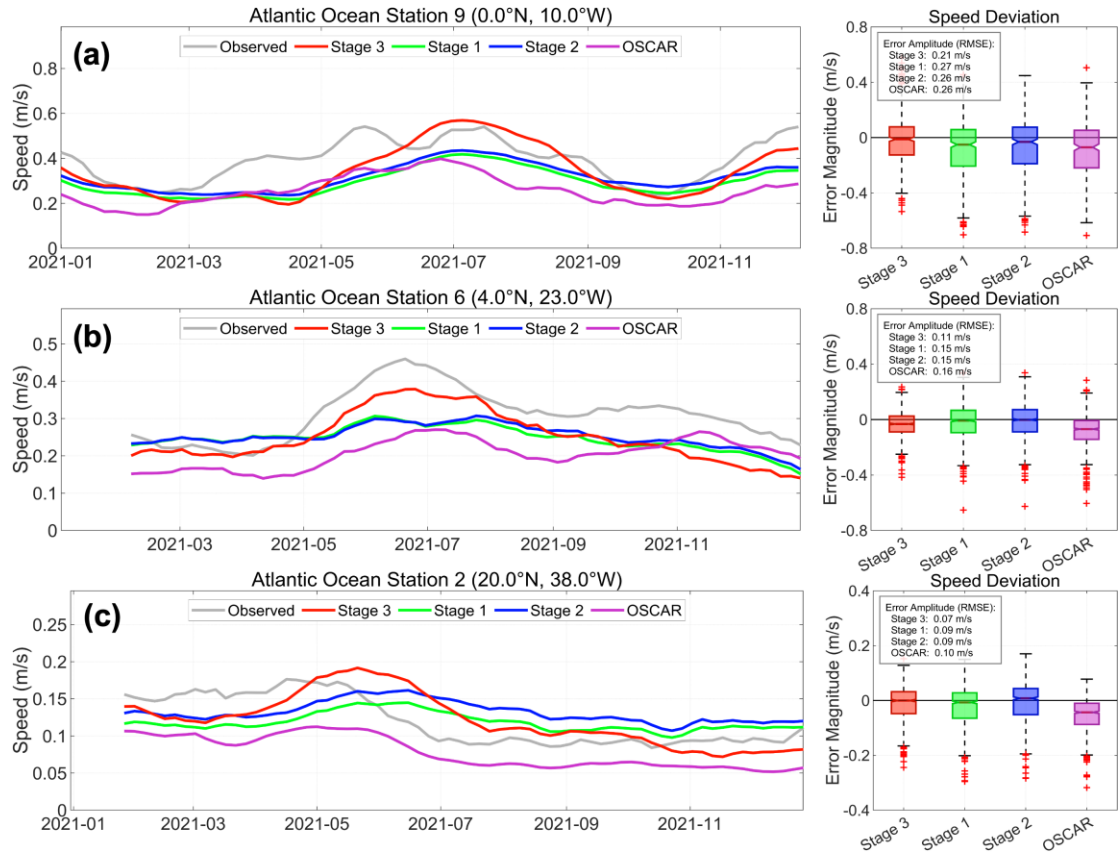


Figure S1. (a) Mean output current field of AC-PIDNN Stage 1 in 2021; (b) Mean output current field of AC-PIDNN Stage 2 in 2021; (c) Final mean output current field of AC-PIDNN Stage 3 in 2021; (d) Momentum balance (Bal) results for Stage 1; (e) Momentum balance (Bal) results for Stage 2; (f) Momentum balance (Bal) results for Stage 3.

To quantify the precise contribution of the physics-informed loss function within the neural network's training pipeline, a dedicated ablation study was conducted utilizing global flow field data from the year 2021. This experiment isolated the different loss constraints to evaluate their impact on both dynamical consistency and observational accuracy. The model was re-trained using the 2021 dataset, and the daily predicted surface currents were extracted across three distinct training phases: Stage 1 (representing the data-loss-only baseline), Stage 2 (hybrid loss optimization), and Stage 3 (the fully physics-constrained final model). By comparing these intermediate outputs against both theoretical momentum balance metrics (Bal) and station-based observational errors, the value added by the physical constraints beyond pure data-driven mapping can be systematically delineated.

A primary objective of integrating a Physics-Informed Neural Network (PINN) architecture is to ensure that the reconstructed ocean surface currents adhere to the governing physical laws of ocean dynamics, rather than merely minimizing the statistical error against remote sensing observations. As illustrated in Figures S1(a)–(c), the annual mean surface current fields generated by the AC-PIDNN model during Stages 1, 2, and 3 exhibit a stark contrast in dynamical consistency when evaluated alongside their corresponding momentum balance residuals (Figures S1(d)–(f)). Because the initial Stage 1 relies exclusively on data loss without any embedded physical constraints, the neural network acts as a standard data-fitting algorithm. Consequently, the output entirely lacks momentum balance correction, leading to severe dynamical inconsistencies where the momentum residuals exceed 100% across the vast

majority of the global ocean regions. Upon introducing a hybridized loss function in Stage 2 that begins blending physical constraints with observational data, a dramatic correction in the flow field's dynamical structure is observed, with the global mean momentum residual dropping precipitously to approximately 51%. In the final Stage 3, the output is rigorously constrained by the complete physics loss. This proper parameterization forces the reconstructed flow field to obey the fundamental equations of motion, vastly improving the momentum balance and reducing the global average momentum residual to a mere 1.2%. This progression unequivocally demonstrates that the physics-informed stages provide a profound improvement extending well beyond initial data fitting. The physics loss serves as a critical mechanism that enforces strict physical consistency, thereby ensuring the hydrodynamic reliability of the reconstructed surface currents.



**Figure S2.** Comparison of the output current fields from the three training stages of AC-PIDNN and OSCAR product velocities against in-situ observational data.

To confirm that this improved dynamical balance translates to real-world accuracy, the outputs

of all three training stages, alongside the standard OSCAR product, were further evaluated against independent in-situ mooring data. Focusing on the velocity time series from successfully matched observation stations in 2021, the progressive incorporation of physical laws was shown to systematically refine the accuracy of the modeled velocities. The average errors were recorded as 0.17 m/s for Stage 1, 0.16 m/s for Stage 2, and 0.13 m/s for Stage 3, compared to 0.17 m/s for the OSCAR product. The data clearly indicate that the Stage 3 output aligns most closely with the in-situ measurements, significantly outperforming the Stage 1 baseline, the intermediate Stage 2 output, and the widely used OSCAR product. Purely data-driven approaches (Stage 1) remain highly susceptible to the inherent noise and spatial gaps present in multi-source satellite altimetry and scatterometry data. By enforcing momentum conservation, the subsequent physics-informed stages effectively filter out dynamically impossible artifacts, preventing overfitting to observational noise and yielding a flow field that is both physically coherent and statistically more accurate. This direct comparison firmly substantiates the superiority and absolute necessity of the physics-informed training stages within the AC-PIDNN framework.



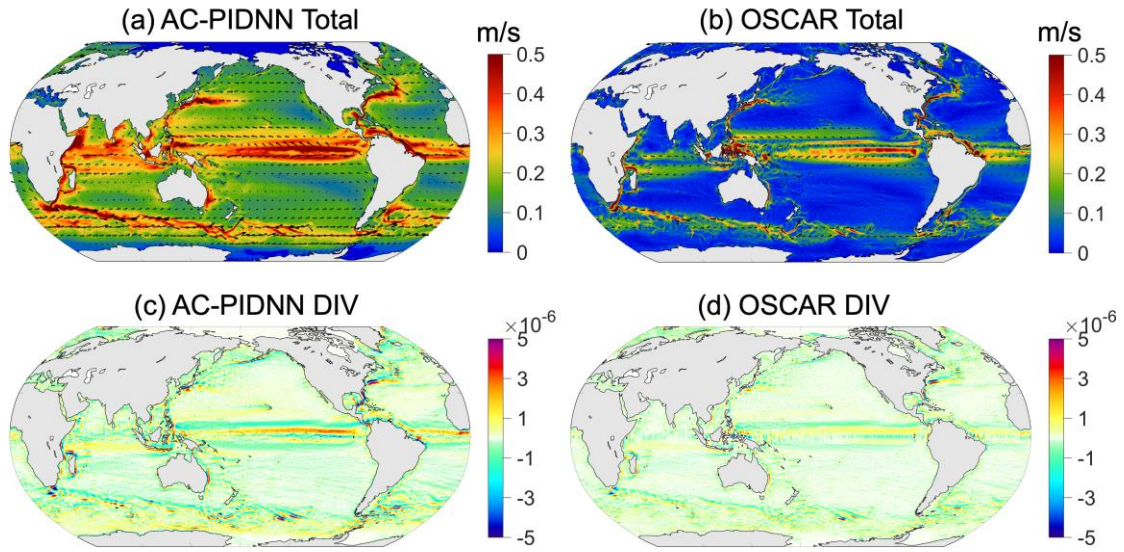


Figure S3. (a) 1993–2022 mean flow field from AC-PIDNN; (b) 1993–2022 mean flow field from the OSCAR product; (c) divergence of the 1993–2022 mean flow field from AC-PIDNN; (d) divergence of the 1993–2022 mean flow field from the OSCAR product.

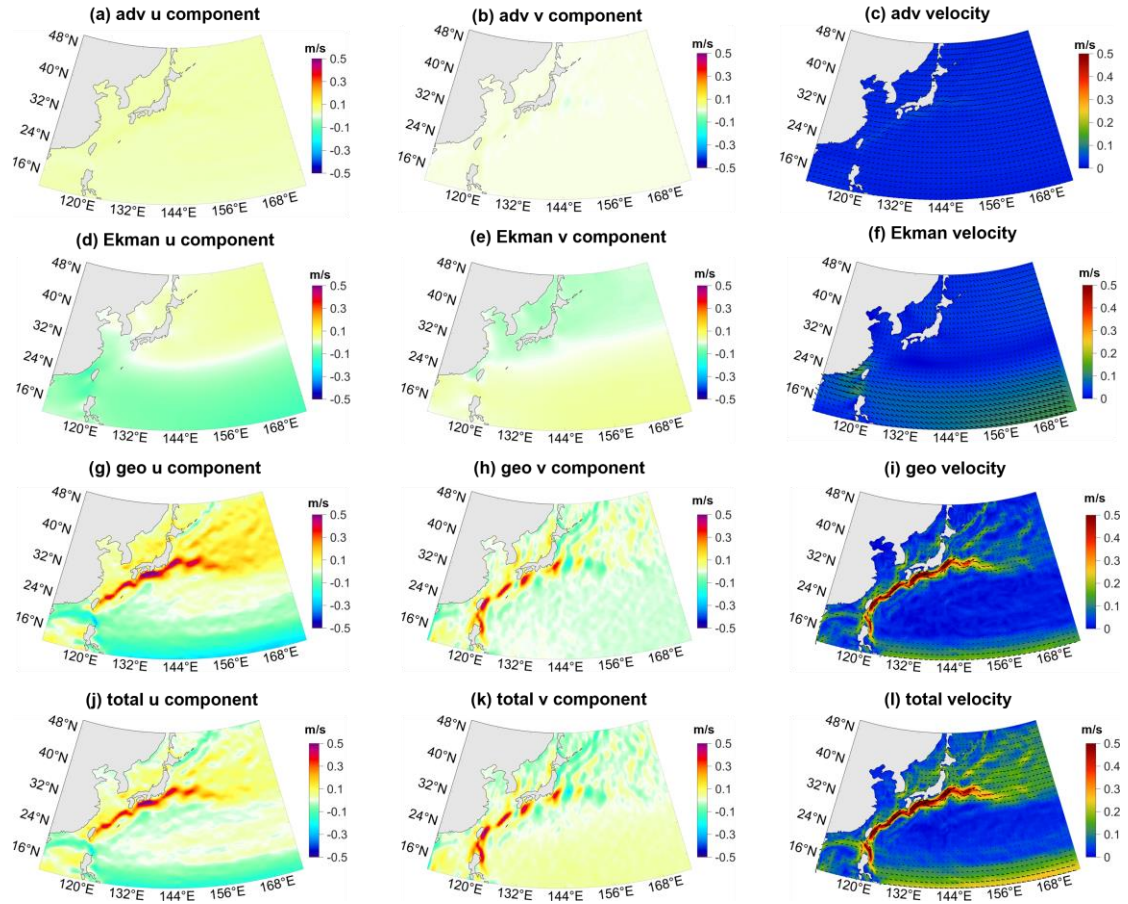


Figure S4: Results of sea surface circulation correction for the Kuroshio Current from 1993 to 2023, using the AC-PIDNN model. Panels (a)–(c) show the u-component, v-component, and flow field of the advective component; panels (d)–(f) show the u-component, v-component, and flow field of the Ekman component; panels (g)–(i) show the u-component, v-component, and flow field of the geostrophic component; panels (j)–(l) show the u-component, v-component, and flow field of the total circulation.

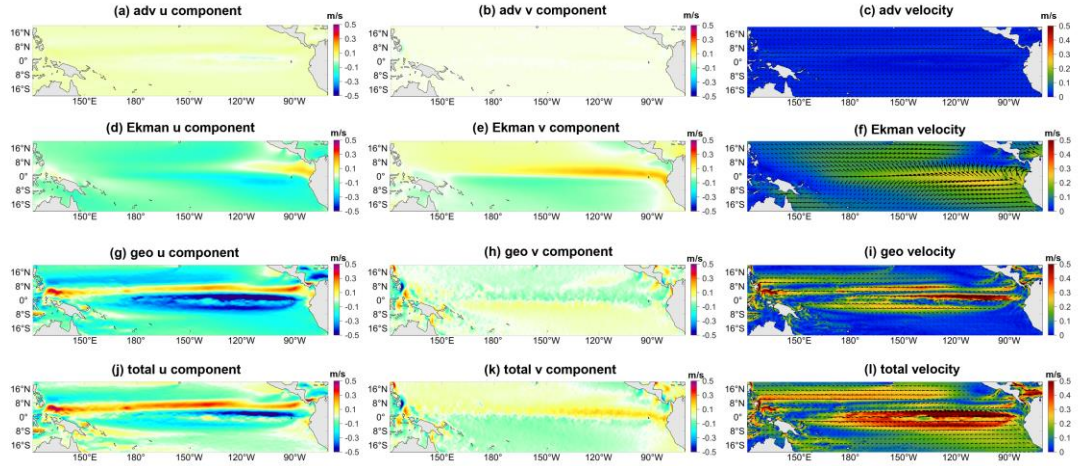


Figure S5: Results of sea surface circulation correction for the Equatorial Region from 1993 to 2023, using the AC-PIDNN model. Panels (a)–(c) show the u-component, v-component, and flow field of the advective component; panels (d)–(f) show the u-component, v-component, and flow field of the Ekman component; panels (g)–(i) show the u-component, v-component, and flow field of the geostrophic component; panels (j)–(l) show the u-component, v-component, and flow field of the total circulation.

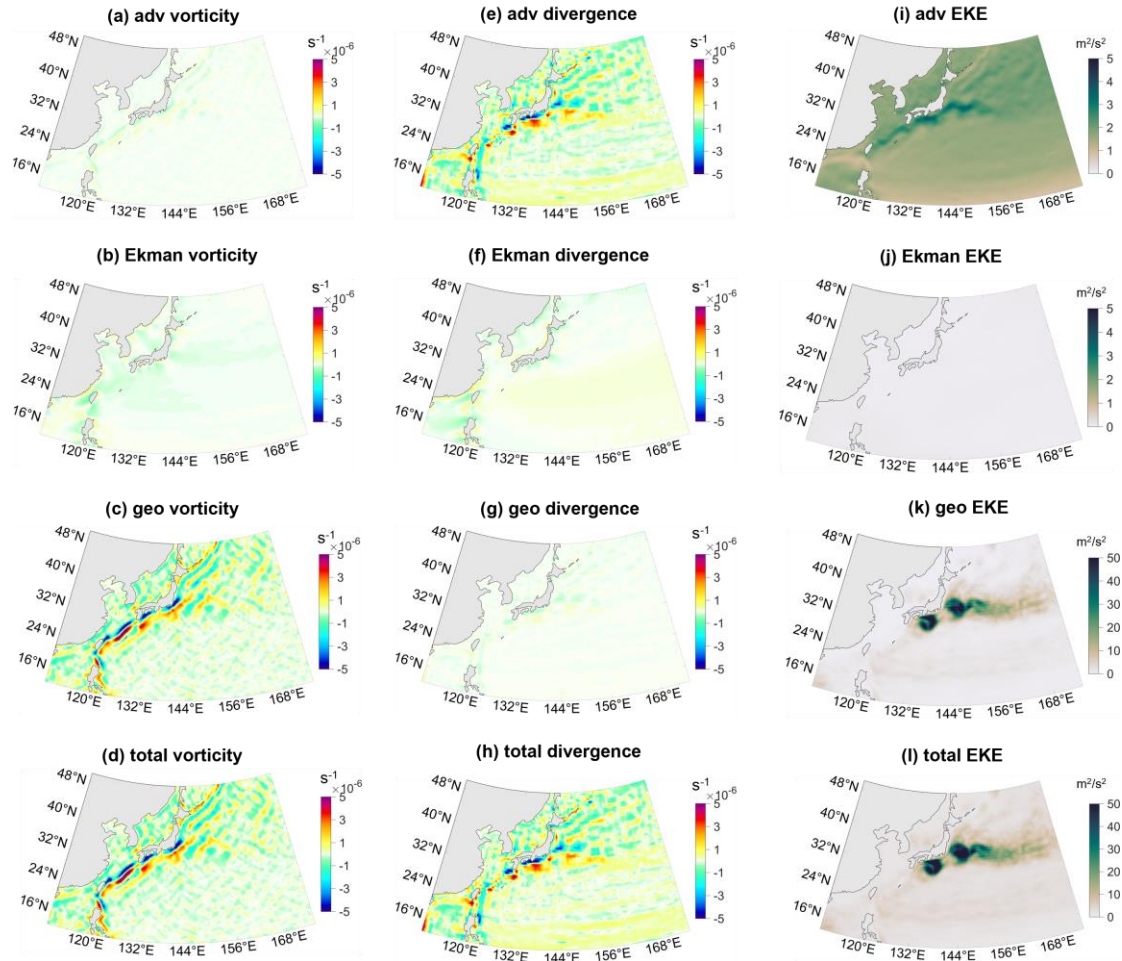


Figure S6: Characteristics of the dynamic parameters of the corrected 30-year average sea surface circulation in the Kuroshio Current. Panels (a)–(d) showing the vorticity of the advective, Ekman, geostrophic, and total components, respectively; panels (e)–(h) showing the divergence of the advective, Ekman, geostrophic, and total components, respectively; panels (i)–(l) showing the eddy kinetic energy (EKE) of the advective, Ekman, geostrophic, and total components, respectively.



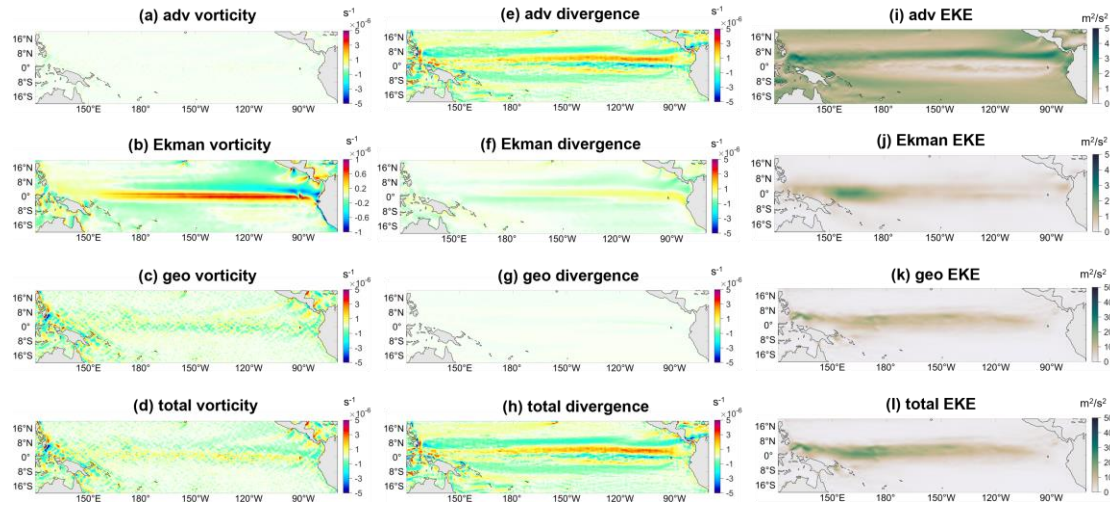


Figure S7: Characteristics of the dynamic parameters of the corrected 30-year average sea surface circulation in the Equatorial Region. Panels (a)–(d) showing the vorticity of the advective, Ekman, geostrophic, and total components, respectively; panels (e)–(h) showing the divergence of the advective, Ekman, geostrophic, and total components, respectively; panels (i)–(l) showing the eddy kinetic energy (EKE) of the advective, Ekman, geostrophic, and total components, respectively.