

Response to Reviewer 1

We sincerely thank you for the time and effort you devoted to evaluating our manuscript. We are grateful for your recognition of the potential interest of our work and for your clear and constructive suggestions, which have provided valuable guidance for our revisions. In response to these comments, we have carefully revised the manuscript and addressed each comment in detail. Our point-by-point responses are presented below in blue, and the corresponding changes have been clearly indicated in the revised manuscript.

Major concern #1: It is not clear what altimeter data they are validating their fitted model on. It appears that it is the same data they used to train the model. In which case, the validation is not a fair comparison with the other models and their superior performance is entirely expected. I would encourage the authors to validate on independent altimetry data which has not been assimilated into any of the models they benchmark against or their model -- SWOT would be extremely useful for this. Without independent validation, I don't feel these results tell us anything.

Response: Thank you for this valuable comment. We agree that independent validation using satellite altimeter data not involved in model construction is highly valuable. However, for the present study, we consider that ten-fold cross-validation uses mutually independent training and validation sets, and that the validation set does not affect the ENOPF results obtained from the other nine subsets used for training.

In Section 3.1, we validate the fitting results against T/P-Jason series satellite altimeter observations. Tidal harmonic analysis was applied to obtain the amplitudes and phase lags of the tidal constituents, which were then used to assess model errors. In response to the reviewer's concern that "In which case, the validation is not a fair comparison with the other models and their superior performance is entirely expected," we have supplemented and clarified the ten-fold cross-validation procedure for the equidistant nodal orthogonal polynomial fitting (ENOPF) method in the revised manuscript. Specifically, the cross-validation uses spatial points as the basic units for partitioning, dividing all spatial locations into 10 subsets. In each cross-validation round, 9 subsets are used to train the ENOPF model, while the remaining subset is used only for validation and evaluation. Thus, in each

round, the training and validation sets are independent of each other, and the spatial points do not overlap; the data in the validation fold do not participate in the ENOPF calculation for that round. After the ten rounds are completed, every spatial point has been used once as validation data. The final cross-validation error is calculated by averaging the vector errors obtained from the ten rounds. As shown in Table 1, the ten-fold cross-validation results are close to the ENOPF results obtained using all track points in Table 4 of the original manuscript, demonstrating the reasonableness of the ENOPF results.

To make this procedure clearer, we have revised Table 1 in the manuscript to present the corresponding 10-fold cross-validation results. Table 1 also responds to the reviewer's comment under "Other miscellaneous comments": "I would be interested to see the actual cross-validation results and not the final outcome. It is interesting that they all converge to orders 4 or 5."

Table 1. Polynomial orders and vector differences from ten-fold cross-validation, together with the

	vector differences of the fitting results							
	M ₂	S ₂	K ₁	O ₁	N ₂	K ₂	P ₁	Q ₁
m	5	4	5	4	4	4	5	4
n	4	5	4	5	5	5	4	5
10-fold-VE (cm)	0.96	0.49	0.42	0.21	0.31	0.31	0.30	0.26
All-track-VE (cm)	0.91	0.44	0.41	0.21	0.29	0.29	0.27	0.24

We also agree with the reviewer's suggestion to use SWOT data for independent external validation. However, the available data from the SWOT Cal/Val span approximately 102 days. Although the sampling interval is close to one day, according to the Rayleigh criterion, a 102-day record is still insufficient to reliably separate all eight major tidal constituents using traditional least-squares tidal harmonic analysis. At present, several methods, such as V_TIDE and MHACS, can be used to process data from SWOT, GFO, and other satellite missions. In future work, we will conduct related studies using results from satellite altimeters other than the T/P-Jason series.

The corresponding changes are on page 8 of the original text, lines 173-174, and line 184.

Table 3. Polynomial orders and vector differences from ten-fold cross-validation and the fitting results of eight tidal constituents (lines 173-174)

	M ₂	S ₂	K ₁	O ₁	N ₂	K ₂	P ₁	Q ₁
m	5	4	5	4	4	4	5	4
n	4	5	4	5	5	5	4	5
10-fold-VE (cm)	0.96	0.49	0.42	0.21	0.31	0.31	0.30	0.26
All-track-VE (cm)	0.91	0.44	0.41	0.21	0.29	0.29	0.27	0.24

We have replaced “satellite altimeter observation data” with “T/P-Jason series satellite altimeter observation data” (line 184).

Major concern #2: Smooth co-tidal maps are not validation and in fact may point to a weakness of the approach. By definition, the authors polynomial fitting approach guarantees this result; however, this feature can lead to neglect of physical features of tidal variability. For example, all of the benchmark models show shifts in the co-tidal lines occurring around islands like Fiji which is entirely expected. By virtue of fitting coarse polynomial features the author's approach neglects this by definition. To me this is a clear weakness of the authors approach and counters the claim that their model can be downscaled to a higher effective resolution. Their claim of increasing the resolution of their approach either needs to be removed or substantiated.

Response:

Thank you for this valuable comment. We agree with the reviewer that "smoothness does not equal accuracy." Using the M2 constituent as an example, we added targeted analyses for Fiji and Hawaii, two regions with pronounced island effects, to examine whether ENOPF alters local phase-lag variations near islands.

The phase gradients and mean absolute errors around Fiji and Hawaii were calculated. The gradient was calculated as follows: as an example, the gradient at point B along the satellite track was calculated, where A, B, and C are three adjacent points, and B is the middle point. In this calculation, Δs is the distance between two points.

$$\frac{\partial f}{\partial x}|_B \approx \frac{1}{2} \left(\frac{f_B - f_A}{\Delta s_{BA}} + \frac{f_C - f_B}{\Delta s_{CB}} \right)$$

In the region near Fiji, ENOPF has the lowest amplitude mean absolute error, at 3.00 cm, which is

lower than those of FES2022 and TPXO10. The gradient mean absolute error of ENOPF is 0.0425 degrees/km, between the values of 0.0312 degrees/km for FES2022 and 0.1141 degrees/km for TPXO10, indicating that it can preserve the principal along-track spatial variations in phase lag.

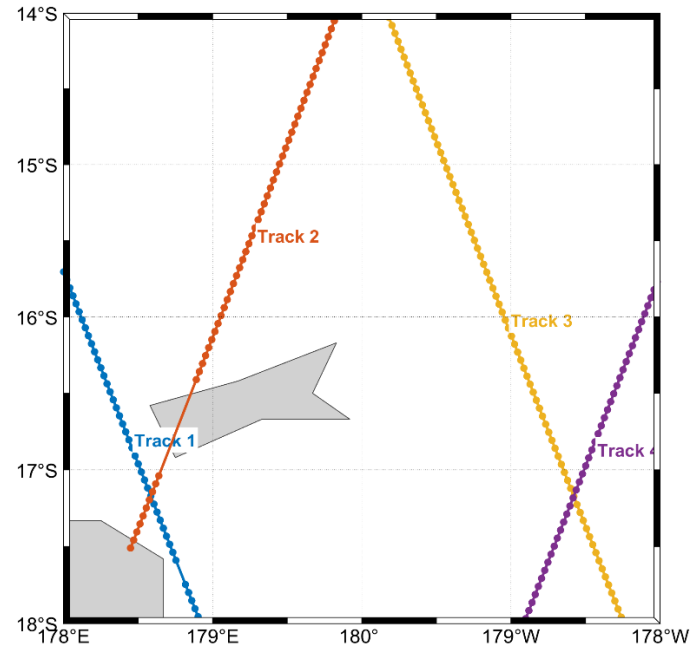


Figure 1. Distribution of satellite tracks around Fiji

Table 2. Results of the Error Assessment for the Surrounding Area of Fiji

Model	Amplitude mean absolute error (cm)	Mean vector error (cm)	Phase-lag mean absolute error (degrees)	Gradient mean absolute error (degrees/km)
FES2022	3.80	2.56	0.57	0.0312
TPXO10	4.06	2.76	1.16	0.1141
ENOPF	3.00	3.29	3.99	0.0425

In the Hawaii region, ENOPF still has the lowest amplitude mean absolute error, at 1.00 cm. However, its mean vector error of 2.09 cm and phase-lag mean absolute error of 9.74 degrees are both higher than those of FES2022 and TPXO10. For M2, which has a period of approximately 12.42 h, 9.7356 degrees corresponds to a time difference of approximately 20.2 min. This error cannot be considered negligible, but it remains below 10 degrees and accounts for approximately

2.7% of one M2 period.

The gradient mean absolute error of ENOPF in Hawaii is 0.0883 degrees/km, slightly higher than the values of 0.0649 degrees/km for FES2022 and 0.0669 degrees/km for TPXO10; all three remain of the same order of magnitude. However, some deficiencies remain in representing the co-tidal lines' details near Hawaii's complex island topography.

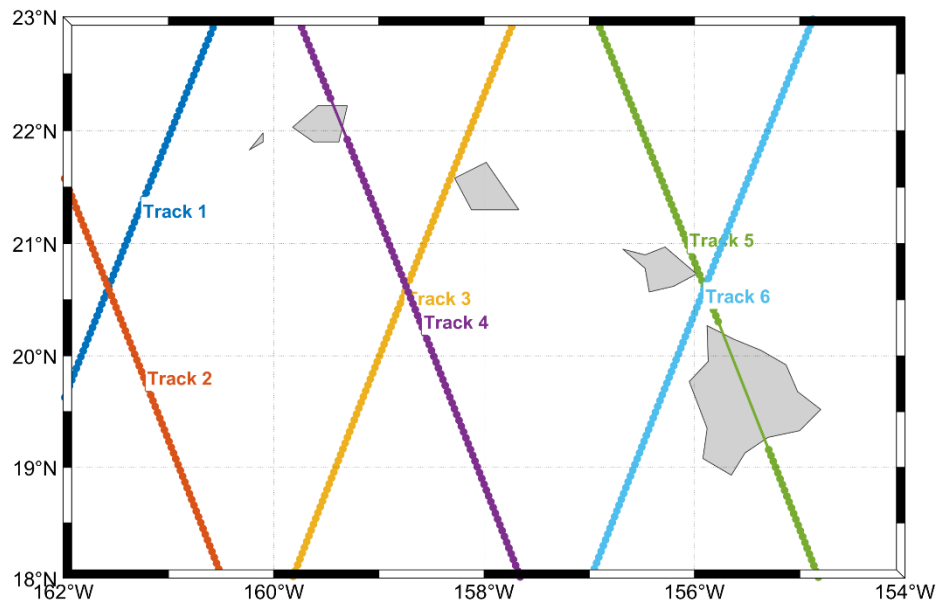


Figure 2. Distribution of satellite tracks around Hawaii

Table 3. Results of the Error Assessment for the Surrounding Area of Hawaii

Model	Amplitude mean absolute error (cm)	Mean vector error (cm)	Phase-lag mean absolute error (degrees)	Gradient mean absolute error (degrees/km)
FES2022	1.39	1.21	2.90	0.0649
TPXO10	1.31	1.05	2.22	0.0669
ENOPF	1.00	2.09	9.74	0.0883

Because the other models are dynamically constrained, they represent changes in co-tidal lines around islands such as Hawaii and Fiji. Since this study does not impose dynamical constraints, relatively large phase-lag errors occur. Future work will investigate the incorporation of dynamical constraints.

We added corresponding content after line 264 in the original manuscript.

Added content:

However, in the vicinity of islands such as Fiji and Hawaii, the ENOPF cotidal lines do not show the local shifts observed in other models. This limitation is likely related to the fact that the current ENOPF is based on spatial polynomial fitting and does not involve dynamical constraints, unlike hydrodynamic or assimilative tidal models. Therefore, although ENOPF provides a smooth reconstruction of the large-scale tidal harmonic constants, it may underrepresent localized tidal variability around islands and complex topography. Future work will focus on improving this aspect by incorporating additional physical constraints (after line 264).

In addition, we agree with the reviewer's comment regarding the statement that the "To me this is a clear weakness of the authors approach and counters the claim that their model can be downscaled to a higher effective resolution." In this method, once the polynomial coefficients of the corresponding orders have been obtained, tidal harmonic constants can be rapidly calculated on an arbitrarily dense output grid by multiplying the coefficients by the basis-function matrix. Therefore, this method increases the spatial sampling density of the output rather than the higher effective resolution. We will correct this statement by replacing expressions such as "downscaled to a higher effective resolution" with "reconstruction on a denser output grid" in the revised manuscript.

The corresponding changes are on p1 of the original manuscript, lines 21-25; p19 of the original manuscript, lines 279-287; p3, lines 71-75.

Furthermore, the ENOPF method offers a simpler and faster method for reconstruction on a denser output grid; subsequently, by extracting the polynomial coefficients, a more detailed dataset with resolution of $1' \times 1'$ can be constructed. This tidal harmonic constant dataset for the Central Pacific Basin offers application value, data support and technical references for tidal dynamics, ocean circulation, and marine engineering projects in the region (lines 21-25).

To improve the capability of deriving tidal harmonic constants in the central Pacific Ocean, this study utilizes observations from the T/P-Jason satellite altimeter and applies the ENOPF method. First, $3' \times 3'$ tidal harmonic constants dataset and cotidal charts are generated; then, by extracting polynomial coefficients, $1' \times 1'$ tidal harmonic constants dataset is constructed. The tidal harmonics

dataset for the Central Pacific Basin offers application value, data support and technical references for tidal dynamics, ocean circulation, and marine engineering projects in the region (Niwa and Hibiya, 2001; Ray and Cartwright, 2001; Kawasaki et al., 2021) (lines 71-75).

As demonstrated in this study, all models show overall consistency in the spatial distribution of tidal constituent amplitudes and the geographical positions of amphidromic points. In the ENOPF method, once the polynomial coefficients of the corresponding orders have been obtained, tidal harmonic constants can be rapidly calculated on an arbitrarily dense output grid by multiplying the coefficients by the basis-function matrix. The tidal harmonics dataset for the Central Pacific Basin offers application value, data support and technical references for tidal dynamics, ocean circulation, and marine engineering projects in the region. We also note that the present ENOPF formulation may underrepresent localized phase variations near islands, such as Fiji and Hawaii, because it does not explicitly include dynamical constraints; this limitation will be addressed in future improvements (lines 279-287).

Other miscellaneous comments

Tables need to have more informative labels. What does bold mean? Is there any notion of uncertainty? Can you compute whether changes are significant?

Response: Thank you for this comment. We have provided further explanations of the bold and other information in the tables. The bold indicates the result with the smallest vector error in the error analysis comparing each model with the T/P-Jason satellite altimeter and tide-gauge results in the Central Pacific Basin (170 °E-230 °W, 30 °S-30 °N). The bold values do not indicate greater statistical significance; they only indicate smaller errors between the model and the observations.

The notion of uncertainty is as follows:

$$SNR_j = \left(\frac{H_j}{1.96s_j} \right)^2$$

where s_j^2 is the variance of the j th tidal constituent. The tidal harmonic constants are considered reliable when SNR_H (signal-to-noise ratio) > 2 (Pawlowicz et al., 2002; Xu et al., 2021).

Table 4. Percentage of points where $SNR_H > 1$ & 2

tidal constituents	Points with $SNR_H > 1$ (%)	Points with $SNR_H > 2$ (%)
M2	99.99	99.98
S2	99.76	99.28
K1	99.20	96.83
O1	98.66	93.71
N2	89.19	61.06
K2	99.80	98.81
P1	96.76	85.49
Q1	47.98	13.09

For Q1, the average amplitude across the entire study area (170°E–230°W, 30°S–30°N) is only 0.88 cm; it is the weakest of the eight major constituents, and points with $SNR_H > 2$ account for only approximately 13.09% of the total area. Compared with the SNR_H of other tidal constituents, the points with $SNR_H > 2$ of the Q1 account for a very small proportion. Further work will focus on improving the accuracy of extracting the Q1 constituent.

The corresponding changes regarding “Tables need to have more informative labels” are on original manuscript p9, line 193, and p11, line 205.

Table 4. The errors of the ENOPF method and the five models for the eight major tidal constituents compared to the satellite altimeter data. Bold values indicate the minimum vector error among all models for each tidal constituent (line 193).

Table 5. The errors of the dataset derived from the ENOPF method and the five models for the eight major tidal constituents compared to the tidal gauges. Bold values indicate the minimum vector error among all models for each tidal constituent (line 205).

Reference:

Pawlowicz, R., Beardsley, B., and Lentz, S.: Classical tidal harmonic analysis including error estimates in MATLAB using T_TIDE, *Comput. Geosci.*, 28, 929–937, [https://doi.org/10.1016/S0098-3004\(02\)00013-4](https://doi.org/10.1016/S0098-3004(02)00013-4), 2002.

Xu, M., Wang, Y., Wang, S., Lv, X., and Chen, X.: Ocean tides near hawaii from satellite altimeter data. Part i, *J. Atmos. Ocean. Technol.*, 38, 937-949, <https://doi.org/10.1175/jtech-d-20-0072.1>, 2021.

Cross-validation: I would be interested to see the actual cross-validation results and not the final outcome. It is interesting that they all converge to orders 4 or 5.

Response: Thank you for this comment. The relevant information is explained in Response to Major concern #1.

The corresponding changes are on page 8 of the original manuscript, lines 173–174.

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We sincerely thank the editor and reviewer for the time and effort devoted to evaluating our manuscript. We have addressed the reviewer's comments point by point and would be pleased to respond carefully to any further comments or suggestions.