Review of the paper entitled Global Thermocline Vertical Velocities: a Novel Observation Based Estimate

1 Major Points

The authors responded satisfactorily to my questions and requests. I am now convinced of the added value of using OLIV3, which is partly based on observations since it uses surface wind-stress provided by the ERA5 reanalysis. I greatly appreciated the rewrite of Section 2.1, which clearly explains the establishment of equation (15). However, for the contribution of the Ekman layer, I have a remark on the treatment of the term $\int_{D_{Ek}}^{0} \vec{\nabla}.\vec{u}_{h,Ek} dz'$ in equation (6).

According to the Ekman theory,

$$\vec{\nabla}.\vec{U}_{h,Ek} = \vec{\nabla} \wedge \left(\frac{\vec{\tau}}{\rho_0 f}\right) \quad \text{your equation (9)}$$

but,

$$\vec{\nabla}.\vec{U}_{h,Ek} = \vec{\nabla}.\int_{D_{Ek}}^{0} \vec{u}_{h,Ek} dz' = \int_{D_{Ek}}^{0} \vec{\nabla}.\vec{u}_{h,Ek} dz' - \vec{u}_{h,Ek}(z = D_{Ek}).\vec{\nabla} D_{Ek}$$

consequently,

$$\int_{D_{Ek}}^{0} \vec{\nabla}.\vec{u}_{h,Ek} dz' = \vec{\nabla} \wedge \left(\frac{\vec{\tau}}{\rho_0 f}\right) + \underbrace{\vec{u}_{h,Ek}(z = D_{Ek}).\vec{\nabla} D_{Ek}}_{Induction\ Term}$$

Finally, to be perfectly precise, your Equation (15) should include what is called the induction term :

$$w_g(z) = \vec{\nabla} \wedge \left(\frac{\vec{\tau}}{\rho_0 f}\right) + \vec{u}_{h,Ek}(z = D_{Ek}).\vec{\nabla} D_{Ek} - \int_{z}^{0} \frac{\beta v_g}{f} dz'$$

In the text, you could mention that the induction term is negligible because $\vec{u}_{h,Ek}(z = D_{Ek})$ is small.

Section 3.2 (Perfect Model Test) : It is unclear to me whether w_g is calculated from the values of v_g and wind-stress of the OGCM model. I think so, but could you clarify, please.

Section 3.5 (Improvement Relative to Ekman Pumping) : How do you explain the poorer performance of w_g compared to w_{Ek} in the NAD, ACC, and Agulhas Current ? Why does the inclusion of the geostrophic component ($-\int_z^0 \frac{\beta v_g}{f} dz'$) degrade the performance in these intense ocean currents ? Would you have obtained better performance by taking $\beta(v_g + v_{Ek})$ rather than βv_g ?

2 Detailed Points

- Line 509 : typos: change "*variaiblity*" to "*variability*"
- Line 537 : typos: change "*except in in regions*" to "*except in regions*"

In conclusion, I accept the publication of this article after taking my comments into account.