Review of the paper entitled Global Thermocline Vertical Velocities: a Novel Observation Based Estimate

1 Major Points

This study proposes the global OLIV3 product of geostrophic vertical velocity in the thermocline, which is derived from ARMOR3D observation-based meridional geostrophic currents and ERA5 surface wind-stress used to derive Ekman pumping. The methodology adopted in OLIV3 relies on the linearization of the vorticity equation, where the vertical stretching term balances the meridional advection of planetary vorticity. The validations of OLIV3 against the perfect OGCM model and the GLORYS12v1 and ECCOv4r4 reanalyses are convincing. OLIV3 captures the interannual variability of tropical and subtropical regions, but fails at fronts and boundary current systems, precisely where subduction and modal water formation occur. Regions between mesoscale structures are also populated by submesoscale structures such as filaments, eddies, whose contribution in terms of integrated vertical transport represents about 50% of the total vertical transport (Klein et al., XX ? I don't remember the date). In short, OLIV3 is efficient in large-scale structures, but has significant shortcomings in crucial regions.

Many products, such as reanalyses, and OGCM, OAGCM models outputs, produce vertical velocities in the thermocline, even at high frequencies. It is therefore important to demonstrate the added value of the OLIV3 database in comparison with these models and reanalyses, simply because models and reanalyses provide the full vertical velocity, which is an important variable for biogeochemistry, for example. Deriving the vertical velocity from the complete vorticity equation or the omega equation shed light into processes driving vertical motion, as well as the balances between these processes. Here the added values of these approaches. Your article should demonstrate the usefulness and applications of the OLIV3 database and not just show that the main large-scale balance lies between meridional advection of the planetary vorticity and vertical w-stretching.

- Thank you for your remark. The primary objective of OLIV3 is to provide an observation-based and dynamically consistent reconstruction of the large-scale geostrophic vertical velocity field, intended to complement existing products derived from OGCMs, reanalysis and observations. Vertical velocity datasets available for the scientific community often differ substantially in their representation of the large-scale circulation as they rely on different variables and data sources, including models, reanalyses, and observations. In particular, we find a substantial discrepancy in the vertical structure of the vertical velocity between OMEGA3D (an observation-based product derived from the omega equation) and two different reanalyses. Even when datasets originate from similar sources, differences in model configuration, spin-up, parameterizations, assimilation methodologies or the equation used to retrieve the vertical velocities can lead to pronounced different estimates of the vertical flow. OLIV3 offers a dataset based on a robust framework (the Linear Vorticity Balance (LVB)) applied to observation-based variables representing the first-order signal for mean states in many regions and for interannual variability globally. This objective precedes the focus on sub- and mesoscale structures, although we fully acknowledge that these processes explain most of vertical kinetic energy in the upper ocean, as demonstrated by Klein et al., 2008.

A key added value of OLIV3 is that it produces a physically consistent large-scale three-dimensional vertical velocity field that reproduces a more realistic baroclinic structure within the thermocline compared with OMEGA3D required to maintain the Sverdrup balance. As a result, OLIV3 should enable the representation of vertical fluxes of passive traces. This aspect is not reproduced by observation-based fields derived from the omega equation available for the community as represented in Fig. 6.

As discussed in Cortés-Morales and Lazar (2024), the LVB framework has indeed notable limitations in regions where the dominant dynamics are sub- and mesoscale, frontal, or strongly ageostrophic (e.g. boundary currents). These limitations are inherent to the scales where the balance holds, which are larger than those characteristic to sub- and mesoscale processes such as filaments, eddies and fronts. Nonetheless, the balance holds well in large-scale regimes where geostrophic dynamics dominate the flow, including key regions such as eastern boundary upwelling systems that play a critical role in defining oxygen minimum zones (OMZs).

Since no "ground-truth" for the vertical velocities exists, by expanding the ensemble of independent reconstructions of the vertical velocity field, each derived from different data and methodologies, we can identify robust large-scale features across methods, and where existing products diverge. In this sense, OLIV3 is not intended to replace high-frequency vertical velocity estimates, but to provide a benchmark based on the available observations that is independent of the numerical biases found in primitive equation models.

The meridional geostrophic velocity (vg) is taken into account in the linear balance equation. I don't understand why the geostrophic vorticity was not conserved in the balance equation (f + ζg) ∂w ∂z = βvg ?

- Thank you for raising this point. In our formulation, the geostrophic relative vorticity term is neglected based on its order of magnitude. In the large-scale circulation regime of interest, characterised by a small Rossby number (Ro $<<$ 1), the terms involving the relative vorticity are typically several orders of magnitude smaller than the Coriolis parameter dependent terms. As discussed in Cortés-Morales and Lazar (2024), this scale separation justifies neglecting the contribution of other terms from the vorticity equation, as its impact is negligible compared to f- and beta-dependent components. However, we acknowledge that this is a good general starting point, but not for a point-to-point assessment, and they may be regions where this assumption is no longer valid and the LVB breaks. For this reason, it is included in the dataset product an additional flag variable indicating the OGCM time-mean relative error and temporal correlation between the total vertical velocity and the LVB-derived geostrophic velocity described in lines 178–180: "*The product is quality-flagged based on the time-mean relative error and interannual correlation coefficient between w_g and w_{tot} in the OGCM perfect model test*". This allows users to identify regions where OLIV3 should be interpreted with caution.

I do not understand how the Ekman pumping is taken into account. In fact, I suspect it is wtot = wg + wek. From my understanding it is about:

Eq. 1 from Review

where h, zgeo and zek are the level of no motion, the depth of thermocline where wg is computed and the depth of the Ekman layer, respectively.

Eq. 2 from Review

This point is essential and must be clarified.

- In this study, we focus on the geostrophic component of the vertical velocity (wg). However, because the total vertical velocity must satisfy the kinematic boundary condition (wtot(z=0) = 0), and considering that the only ageostrophic component of the vertical velocity is the Ekman pumping (wek), wg is necessarily balanced by wek at the surface, i.e., wg(z=0) = -wek(z=0).
  Following your comment, we have revised subsection 2.1 in the Methodology and Data section and added a new Appendix A to provide a more detailed description of the Ekman pumping contribution. In particular, we now explicitly justify why the boundary condition is imposed at the surface rather than at the base of the Ekman layer under a beta-plane approximation, understood as beta that the meridional derivative of f exists locally, following in the discussion in Pedloski (1979), where it is theoretically demonstrated that the divergence of the geostrophic horizontal flow is not zero on a beta-plane, and this justifies the assumptions in our approach.

It is mentioned in the conclusion that total meridional velocities (vg + vag) and additional terms from the vorticity equation, such as the horizontal advection of relative vorticity should be incorporated. In some way, you have already incorporated an ageostrophic component of the current with Ekman pumping. By introducing vag in equation f ∂w ∂z = β(vg + vag), what do you expect on OLIV3 performances ?

- We expect that including the ageostrophic meridional velocity in the balance would extend the regions where the LVB framework can describe the vertical flow. Because the thermocline circulation is largely geostrophic, the inclusion of the ageostrophic meridional velocities is not expected to substantially improve the OLIV3 accuracy in these. However, including the total meridional velocity could extend the accuracy of the vertical flow, closer to the intergyre region for example. The largest potential benefit is expected below the thermocline, where the existence of a level of no motion makes the vorticity balance more sensitive to ageostrophic contributions (Cortés-Morales and Lazar, 2024). Although it is true that we have already included the ageostrophic component with Ekman pumping, we have included its contribution throughout the Ekman layer, but not its contribution at each level within it and how it affects the total vertical velocity. Furthermore, although geostrophic circulation dominates horizontal circulation in the thermocline of tropical and subtropical gyres, this is not the case in regions such as the WBC (Cortés-Morales and Lazar, 2024), where the ageostrophic component has a non-negligible role. Therefore, Ekman pumping is not the only ageostrophic component of vertical circulation.

How do you intend to compute vag ?

- In the ocean interior, the flow is mainly geostrophic. However, it is not the case in the ocean interior. Using total currents at these levels from gridded observation-based products, such as GLOBCURRENTS (https://doi.org/10.48670/mds-00327) or AGESC-Med (https://doi.org/10.1016/j.dib.2023.109804) in the Mediterranean Sea and in-situ measurements such as the ones contained in the Global Ocean-Delayed Mode in-situ Observations of surface and sub-surface ocean currents product (https://doi.org/10.17882/86236) could add additional terms to the geostrophic velocities from OLIV3.

To have consistency between vg and vag, isn't it better to use v and also the vorticity ζ from a reanalysis ?

- Thank you for you suggestion. Using reanalysis for the total meridional velocity and relative vorticity is a valid option. However, for OLIV3 we deliberately chose an observation-based reference (ARMOR3D) available for the scientific community. OLIV3 is intended to be an observation-based product.

Figures 4, 5, and 6 show OLIV3, GLORYS12, ECCOV4 and OMEGA3D. However, it is the differences between OLIV3 and these other products that are discussed. These differences are very difficult to see. Please provide Figures illustrating these differences.

- We thank the reviewer for this comment. In the revised manuscript, we have added new figures that explicitly show the differences between OLIV3 and the three comparison products (GLORYS12, ECCOv4, and OMEGA3D) and among them. These new panels are now included alongside the original fields in the new Figures 5-8, allowing the spatial patterns of agreement and disagreement to be visualized much more clearly.

It is better to use the vertical velocity in m/day rather than in m/s.

- All figures and text unit references have been changed from m/s to m/day.

2 Detailed Points

- Line 76 : typos: change "gesotrophic" to "geostrophic"
    - Corrected
- Line 98: "local mass balance between meridional divergent flow and an opposing vertical convergence". Ok but what equation 1 shows is a balance between vertical convergence and meridional advection of planetary vorticity.

- Changed in line 108-109 by: "*Physically, Eq. 1 expresses the balance between the meridional transport of the planetary vorticity by geostrophic flow and the vortex stretching induced by the vertical motion.*"

- Line 101: This sentence is confusing because the horizontal geostrophic flow is nondivergent. Please reformulate.
  - Thank you for pointing this out. We have thoroughly revised Section 2.1 and added a new Appendix A to improve clarity regarding the divergence of the geostrophic flow and the role of Ekman pumping. In particular, we now explicitly discuss that while the horizontal geostrophic flow is nondivergent on the f-plane, this is not the case on the β-plane (Pedlosky, 1996).
- Line 106-107: The Ekman pumping wek occurs at the Ekman pumping depth (Dek =0.2∗√τ/f, Li et al., 2021; GRL). So a vertical profile of wek is often prescribed fromthe surface, where w = 0, to z = zek, where w = wek, to z = 2Dek, where w = 0. So w = wek at z = 0 is not a good surface boundary condition. Please correct.
  - Thank you for the comment. We have updated section 2.1 including a discussion following previous literature about the need to define Ekman pumping at the surface in a beta-plane (where the geostrophic divergence is not zero) instead at the Ekman depth. See also our response to the general comment above.
- Line 133: Ekman pumping is not clearly shown in Equation 2. See remark in the "Major Points" Section.
  - Changed in the update of section 2.1.
- Line 135: Here again it is not wg because it includes wek. This is confusing because, as said lines 131-132, the product w of OLIV3 has two components, which are wg and wek. Please clarify this point.
  - We have improved the explanation to avoid misunderstanding.
- Line 150: Isn' it better to calculate vg from the thermal wind equation? Based on pressure, the result is often noisy, unless the pressure is first smoothed. In this case, the filtering procedure should be mentioned.
  - We agree with the reviewer on the possibility of retrieving the meridional geostrophic velocity from the thermal wind relation. However, this approach requires computing vertical derivative of the velocities, implying the depth integral of the temperature. We preferred to not add more steps to the computation to avoid the propagation of errors.
- Line 153: The reference Jean-Michel et al., 2021 is not adequate. You cannot use the first name of the authors in references. Please correct.
  - Thank you for noticing. In the referenced publication, the First and Family Names are inverted. We have corrected the citation in line 196 and the references list in line 763.
- Line 165: Omega-equations need not only surface momentum and heat air-sea fluxes, but also fluxes in the ocean. Where do these fluxes come from ?
  - The reviewer is correct that the omega-equations require not only surface air–sea fluxes but also fluxes within the ocean interior. In the OMEGA3D product, forcing terms are computed from ARMOR3D potential density and

geostrophic velocity fields as well as ERA Interim atmospheric reanalyses (Buongiorno Nardelli, 2020). It is mentioned in line 210: *"... and ERA-Interim (Dee et al., 2011) surface air-sea fluxes."*

- Line 178: The equator band (5S/N) is large. Geostrophism can be applied from 2S/N degrees, and even 1S/N degree. For example see Dourado and Caniaux, JGR, 2001 (their Figure 4).
  - Thank you very much for the suggestion and reference. We understand that in theory the geostrophic approximation can be extended closer to the equator. However, we have selected the equatorial mask (5°S–5°N) to remain consistent with the ARMOR3D product (Mulet et al., 2012). From ARMOR3D product QUID (https://documentation.marine.copernicus.eu/QUID/CMEMS-MOB-QUID-015-012.pdf): *"At the equator, the thermal wind equation is no more valid because the Coriolis parameter f is zero. Therefore, the method is adapted between 7°S and 7°N: For the zonal component, the velocities are estimated with a second order differentiation (Picaut & al, 1989)."*
- Line 182-184: Why was the isopycnal level σ26 chosen? How does it compare to the the mixed-layer depth? Why not choose the mixed-layer depth ?
  - Thank you for this question. The isopycnal level σ26 was chosen as a representative depth within the thermocline, where the LVB approximation is valid. In Cortes-Morales and Lazar (2024), we have demonstrated that the σ26 is a representative example of the thermocline. As you can observe in Figure 7 from the same paper, the LVB approximation holds valid below the MLD, so it could be also possible to show this level with the same qualitative properties. This is clarified in lines 226-228: "*This isopycnal level was chosen to assess the vertical velocity estimates across most of the extension of the global subtropical gyres, while maintaining a focus on thermocline dynamics, where the LVB framework performs best (see CM24 for the North Atlantic Ocean).*"
- Line 188-189: Explain why Figure 1 emphasises the role of atmospheric forcing as the primary driver of vertical flow within the upper ocean. Are you implying that the Sverdrup balance can be used to obtain a good estimator of vg ?
  - Thank you for the comment. In the revised Figure 2 (previous Figure 1), we have included an additional panel (b) showing the Ekman pumping at the ocean surface. This allows a direct comparison between the vertical velocities at the ocean surface (Ekman pumping) and those in the ocean interior (OLIV3 at sigma26). The overall agreement in the large-scale patterns of upwelling and downwelling between the two levels suggests the dominant role of the Ekman pumping to the vertical velocity in the ocean interior compared with the divergence of the horizontal geostrophic flow.
  - Regarding the Sverdrup balance, we do not intend to imply that it provides a direct estimator of the geostrophic meridional velocity. Rather, the relationship is conceptual: the computation of OLIV3 can be interpreted as the indefinite integration of LVB (Sverdrup balance being the definite integration of LVB), such that the geostrophic vertical velocity at a given

    depth represents the fraction of the atmospheric pumping that is not evacuated by the horizontal divergence above that depth. When the geostrophic vertical velocity at a given depth is effectively null, the divergence of the above horizontal flow fully evacuated the atmospheric input, therefore assuming the Sverdrup balance describes the ocean dynamics in the location up to the given depth. This is now clarified in lines 390-394:" *When the LVB holds, geostrophic vertical velocities in the ocean interior can be interpreted as the residue of the evacuation by meridional transport of the vertical mass flow input from the layer above. If the geostrophic vertical velocity at a given depth is effectively negligible, the divergence of the horizontal flow fully compensates the wind driven divergence above this level, implying that the Sverdrup balance adequately describes the ocean dynamics down to that depth.*"

- Line 194-195: This aspect is an issue because we do not see the point of using LVB. Please identify and discuss the missing processes in the LVB to correctly represent the frontal dynamics.
    - Thank you for the comment. In the western boundary currents (WBC) and other frontal regions, the LVB is no longer a good approximation of the vorticity balance, as shown in Cortés-Morales and Lazar (2024). Previous studies focused on the Depth-Integrated Vorticity Balance equation have demonstrated that the bottom pressure torque (BPT) effectively balances the barotropic planetary vorticity advection, in the WBC, with the wind forcing being negligible (e.g., Hughes and de Cuevas 2001; Gula et al. 2015; Schoonover et al. 2016) as indicated in lines 380-382. Additionally, nonlinear advection of relative vorticity contributes substantially to closing the vorticity budget, as further supported by OGCM analyses in the North Atlantic (Cortés-Morales, 2024, thesis). However, the LVB performs well in regions such as the eastern boundary upwelling systems (EBUS) as indicated in lines 559-561. It is important to note that the LVB captures large-scale patterns, but it cannot resolve finer-scale dynamics, which are dominated by nonlinear and ageostrophic processes (Cortés-Morales and Lazar, 2024).
- Line 196-197: Please show a Figure of Ekman pumping.
    - Thank you for the suggestion. We have included a panel (b) in Figure 2 (previous Figure 1) showing the Ekman pumping.
- Line 230: Equation 4. Using $\sigma_{27}-\sigma_{MLD}$ makes difficult to understand the following discussion, because we do not know the sign of this difference. Then the speech is difficult to follow. I suggest the metric $\frac{\partial w_g}{\partial z}\Big|_{z=MLD} - \frac{\partial w_{tot}}{\partial z}\Big|_{z=MLD}$ instead, normalized or not.
    - Thank you for pointing out. We have revised Equation 13 (previous Equation 4) to use the vertical distance between isopycnal surfaces (in meters) rather the difference in density (sigma27-sigmaMLD) and we have change the name to "diapycnal gradient". In this formulation, we consider only the magnitude of the distance, not the sign, focusing on regions where the mixed layer is shallower than sigma 27. In this way, negative values indicate

a decrease in magnitude with depth, while positive values indicate an increase. The revised equation and comments are updated in line 275: "*Negative values indicate a decrease in magnitude with depth, while positive values indicate an increase.*"

- Line 245-250: This shows the limits of the method in frontal regions. Even in coastal regions, Ekman pumping fails to capture vertical transport of physical and biogeochemical tracers.
    - We agree with the reviewer that OLIV3 has substantial limitations in the description of the vertical velocities in frontal and coastal areas. The LVB cannot reconstruct completely the vertical flow in these regions because the geostrophic component is not the dominant contributor to the vertical velocity in these areas. However, large uncertainties between the various references still exist. To help users assess the reliability of OLIV3, we have included an additional flag variable indicating the OGCM time-mean relative error and temporal correlation between the total vertical velocity and the geostrophic vertical velocity computed from the depth-integrated LVB as described in lines 178–179: "*The product is quality-flagged based on the time-mean relative error and interannual correlation coefficient between w_g and w_{tot} in the OGCM perfect model test*". This allows users to identify regions where OLIV3 should be interpreted with caution.
- Line 272-273: I don't understand why a downward decrease of wg. I would instead expect a positive vertical gradient. I am having trouble following the discussion about the vertical gradient of w, because the sign of (σ27 − σMLD) is unclear.
    - Thank you for pointing this out. We recognise the unclear wording. We have changed the sentence in the lines 316-319 to clarify the discussion of the diapycnal gradient: "*Note that the diapycnal gradient of the time-mean total vertical velocity is almost everywhere positive (non-dotted areas), indicating a decrease in the magnitude of the vertical velocity toward the base of the thermocline. This structure is consistent with a baroclinic velocity field, generating a tachocline…*" . This is also included in lines 379-382:" *Particularly, western boundary current systems correspond to regions with large errors in the geostrophic LVB-derived vertical velocities (hatching in Fig. 5a). In these regions, additional terms of the vorticity equation, such as the bottom pressure torque, close the vorticity budget (e.g. Hughes and De Cuevas, 2001; Gula et al., 2015; Schoonover et al., 2016).*"
- Line 330-331: Reanalyses are significantly affected by spin-up effects, primarily vertical velocity. This is why incremental analysis update techniques are used in data assimilation procedures. Consequently, how much confidence can we place in such reanalysed vertical velocities, given that they are partially affected by unphysical spurious effects ? In other words is it reasonable to use them as w-references ?
- Line 335: Reanalyses are significantly affected by spin-up effects.
- Line 428: Not good due to spin-up.

- Response for Lines 330-331, Line 335 and Line 428 comments. Thank you for raising this these points. Although the reanalyses present uncertainties, reanalyses such as GLORYS12 and ECCOv4 are widely used in the oceanographic community (e.g., Wunsch, 2011, Gray and Riser, 2014, Thomas et al., 2014, Liao et al., 2022). Additionally, the comparison between different datasets with different input sources and methodologies allows us to identify robust large-scale features that are consistently represented across products. This multi-dataset comparison provides a reliable baseline for validation, even in the presence of individual dataset uncertainties and spin-up artifacts. Furthermore, all these issues are an additional motivation for using OLIV3.

- Line 348-349: I don't understand this sentence. I would say that the geostrophic vertical velocity in the ocean interior results from the convergence/divergence of the Ekman drift.
  - Thank you for the comment. As discussed in Section 2.1, the geostrophic vertical velocity at a given depth is determined by two contributions: the Ekman pumping at the ocean surface and the β-plane divergence of the geostrophic flow in the interior.
- Line 355: Figure 5. Sorry but I am lost with vertical gradient expressed in ms^{-1}/kgm^{-3} . Where does $\sigma 27$ fit in relation to σMLD ? I suggest expressing this gradient in $day^{-1} = mday^{-1}/m$.
  - Thank you for the suggestion. We have revised the calculation of the vertical gradient, now "diapycnal gradient" in Figure 6 (previous Figure 5) by using the vertical distance between isopycnal surfaces (in meters) rather than the difference in density ($\sigma 27 - \sigma MLD$), as you suggested. Only regions where the mixed-layer depth (MLD) is shallower than $\sigma 27$ are considered, ensuring that the gradient reflects the vertical structure of the thermocline consistently. We have changed the figures and text according to the new metric.
- Line 375-376: Be careful $w(z = 0) \neq wek$.
  - With the update of Section 2.1 and the addition of Appendix A, we are confident that this affirmation is correct.
- Line 387: Change Fig.5b to Fig.6b.
  - Thank you for noticing it. Changed
- Line 391-393: Arbitrary conclusion at first glance (Fig 6). Make difference maps.
  - We have included the difference map in the new Figure 8 to add robustness to our discussion.
- Line 446: OMEGA3D also integrates vertical stratification.
- Line 450: OMEGA3D is a physical investigating tool because it is based on the destruction of the thermal wind balance by current and turbulence.
  - Thank you for these two comments. To clarify, we have updated the lines 487-488: "*In contrast, OMEGA3D employs the omega equation, which, although it also requires vertical integration, explicitly includes second-order vertical derivatives and horizontal derivatives of w.*"

- Line 527-530: If I am a biogeochemical scientist, or physicist who wants to estimate modal water production, what is the benefit of using OLIV3 rather than a reanalysis ? Sorry, I'm not convinced, but I would like to be.
    - Thank you very much for your question. OLIV3 provides a robust description of the vertical flow in the open-ocean large-scale subtropical downwellings and tropical upwellings, in particular its interannual variability. Therefore, if one desire to study more water formation variability, we recommend to use OLIV3 in priority to any other existing global product since w variability in mode water regions is likely completely dominated by its geostrophic component (if we trust the comparison showing very good correlation between wg and wtot in our reference OGCM, Figure 4b). Compared with reanalysis, OLIV3 is a tool based on observations without being affected by all the biases of reanalyses products (e.g. spin up effects), as the reviewer commented. While studies such as Bellacicco et al. (2025) use OMEGA3D to estimate the biological carbon pump, OLIV3 offers a complementary dataset with a more physically consistent vertical structure than OMEGA3D. Again, our aim is to provide the community with an additional variable computed from and independent input and a different methodology to what it is available for the community at the moment.
- Line 532: Before incorporating non-linear processes, integrate before the total meridional velocity and vorticity.
    - Changed "full" by "total" in line 578 to improve understanding.

In conclusion, I request substantial changes, particularly on the interest of using OLIV3, and clarifications on the incorporation of Ekman pumping in Equation 2, and the physical interpretation of this equation balance.

- We thank the reviewer for the revision and for highlighting all these key points. In response, we have made substantial revisions to the manuscript to clarify the interest and added value of OLIV3, the relationship between Ekman pumping and the geostrophic vertical velocities at the ocean surface and interior, and the metrics used for the validation of the product. In addition to addressing the specific issues in their locations in the text, some changes have been applied to the abstract and the conclusions section to reflect them.
- We also realised that the manuscript repeatedly referred to our previous study (Cortés-Morales and Lazar, 2024). To lighten the text, we now refer to this wors as CM24 throughout the manuscript.