Responses to reviewer 1#'s comments point by point

MS No.: essd-2025-384

Title: A Black Hole Eddy Dataset of North Pacific Ocean Based on Satellite Altimetry

Author(s): Fenglin Tian et al.

Dear Reviewer,

We highly appreciate the detailed and valuable comments of the referee on our manuscript entitled "A Black Hole Eddy Dataset of North Pacific Ocean Based on Satellite Altimetry (ID: essd-2025-384)". These comments are all valuable and helpful for revising and improving our paper, as well as providing important guidance for our research. In the past few days, we have referred to the comments and improved the paper.

As follows, we would like to clarify some of the points raised by the Reviewer. The original comments begin with "**Comment**" and are quoted in italicized font, the responses begin with "**Response**" in normal font, the original sentences and phrases are in blue letters, the revised sentences and phrases are in red letters, and the line number in the revised manuscript is highlighted in yellow. We appreciate the Reviewer's warm work and taking the time to review the manuscript, and we hope that the corrections will meet with approval.

Yours Sincerely,

Fenglin Tian, Yingying Zhao, Lan Qin, Shuang Long, Ge Chen

2025-8-6

Comment:

This dataset paper presents an ambitious attempt to detect materially coherent Lagrangian eddies (so-called Black Hole Eddies, BHEs) in the North Pacific using a GPU-accelerated implementation of a geodesic eddy detection algorithm. The work contributes to the growing demand for Lagrangian eddy datasets derived from altimetry, with relevance to ocean transport, climate dynamics, and eddy-resolving model evaluation.

The authors present a new dataset (BHE v1.0) of 18,387 long-lived, materially coherent eddies in the North Pacific (1993–2023), derived via a GPU-enhanced version of the null-geodesic detection method for coherent Lagrangian vortices. They compare BHEs against Eulerian and RCLV-based eddy atlases, highlight eddy coherence via Lagrangian particle advection, and quantify zonal and meridional transports. They also report a subset of "Naked BHEs" not detected by conventional methods.

While the technical ambitions of this study are commendable, the manuscript falls short in multiple key areas. The detection framework, while important, lacks transparency, and several scientific claims are overstated without support from independent benchmarks or statistical uncertainty. The language and structure are often difficult to follow, further obscuring key methodological and conceptual points. With improvements in algorithmic clarity, quantitative rigor, and language polish, particularly the very concerning use of AI writing tools, this paper could contribute to ESSD and the broader eddy-resolving community.

Response:

Thank you for the thoughtful and constructive feedback. We appreciate the recognition of the technical ambition and the potential contribution of our dataset to the Lagrangian eddy research community. At the same time, we acknowledge your concerns regarding algorithmic transparency, scientific overstatement, and clarity of language. We hope that the revisions address your concerns and improve the overall quality of the manuscript. Please find our detailed responses to the specific comments below.

General Comments:

Comment:

1. Inadequate Algorithm Description and Lack of Code Transparency

The paper presents a GPU-accelerated version of the null-geodesic BHE detection method but does not provide sufficient technical detail to reproduce the dataset. For example:

① *There is no publication or sharing of the source code, despite the centrality of the algorithm to the study.*

② *The CUDA kernel logic, grid configuration, and interpolation choices (e.g., cubic B-splines) are only briefly mentioned and not documented in usable form.*

③ *The description of the numerical solver for null geodesics is too high-level and lacks algorithmic pseudocode, flowcharts, or performance benchmarks beyond one test case (January 1, 2021).*

①**Response:** We acknowledge your concern regarding the unavailability of the full source code. At this stage, the complete implementation of the BHE detection algorithm cannot be made publicly available due to institutional intellectual property restrictions. Additionally, the algorithm is still undergoing internal optimization and will continue to evolve as we expand the dataset.

However, to support reproducibility and public use of the dataset, we have provided open-source script for loading and visualizing the published data products. The script is included in the data repository [https://doi.org/10.5281/zenodo.15597447], allowing users to explore the results and apply the dataset to their own research. Furthermore, we have added detailed algorithmic pseudocode to Appendix A, which outlines the numerical procedure used to extract BHEs via null geodesic calculation. This addition aims to make the implementation transparent and reproducible for users.

②**Response:** Thank you for pointing out the lack of details regarding the CUDA kernel logic, grid configuration, and interpolation choices. We have added two schematic diagrams in the revised manuscript that illustrates the grid configuration (Fig. 2) (Please see Line 277-280) and CUDA kernel workflow (Fig. 3) (Please see Line 326-327) in detail.

As illustrated in Fig. 2, we enhanced the accuracy of the simulation by densifying the original velocity field grid with a resolution of 1/4° by a factor of 8. Figure 2 shows red solid nodes in the center, representing the main grid points. These are the input velocity field data points and serve as the primary computation locations for the Cauchy-Green strain tensor field. The green points surrounding them are the auxiliary nodes post-densification, and the black points correspond to the auxiliary grid point x_0. Following densification, we calculate the Cauchy-Green strain tensor for both the main grid points and auxiliary nodes. This detailed grid configuration facilitates a more profound understanding and enhanced accuracy in simulating the dynamics of BHE.

Figure 3 shows CUDA thread allocation logic. once the size of the initial grid($(m \times k) \times (n \times k)$) and the number of threads per Block in the GPU $(m \times n)$ are inputted, the size of the Block to be allocated for each

Grid $(k \times p)$ can be determined (Fig. 3).

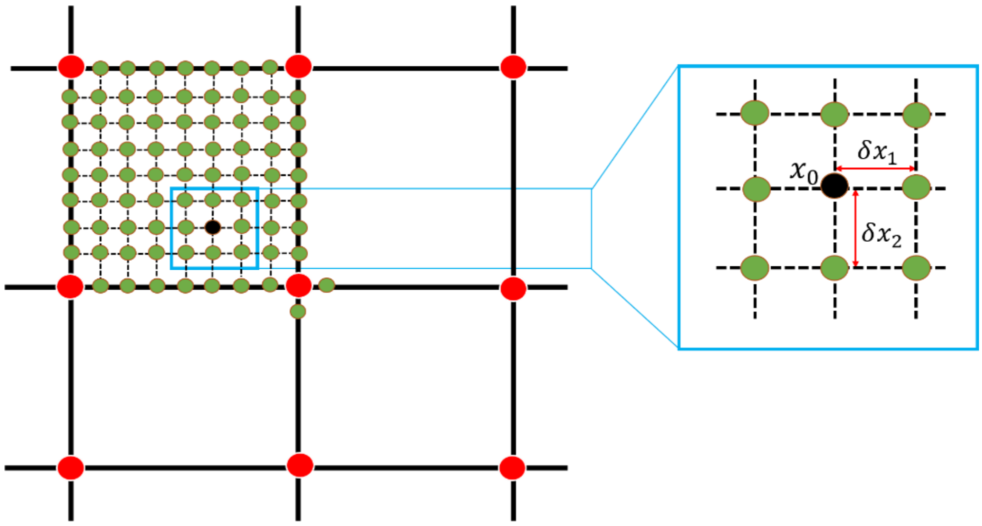

Figure 2. Schematic diagram of the spatial relationship between a main grid point and its surrounding auxiliary points in the Cauchy-Green strain tensor field. The black dot represents an auxiliary grid point x_0, δx_1 and δx_2 denotes the distance between the auxiliary grid point (green) and the main grid point (red).

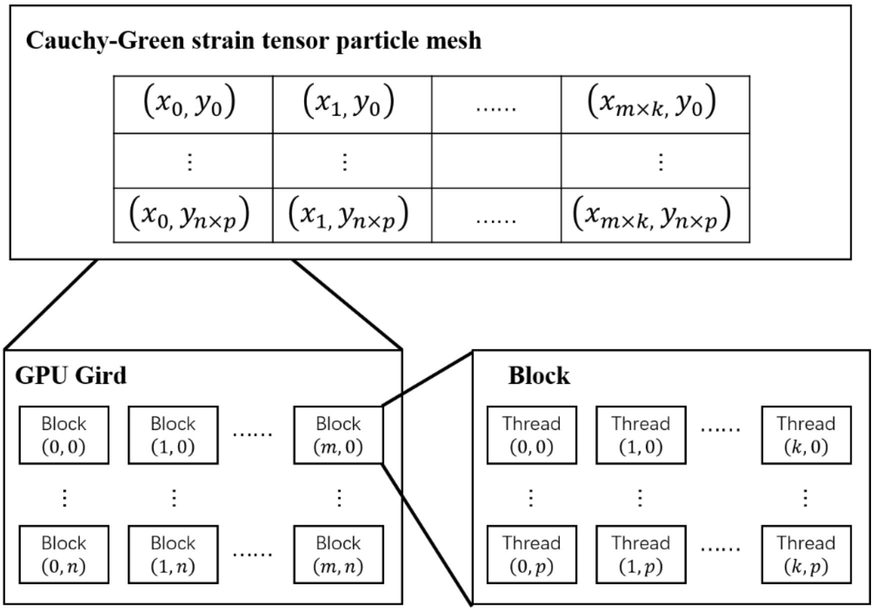

Figure 3. Thread allocation diagram.

Regarding the interpolation method, we have clarified that the cubic or B-spline interpolation is applied specifically in the computation of the Cauchy-Green strain tensor field, which forms the basis for variational vortex boundary detection. This interpolation ensures smooth and accurate estimation of the deformation gradient ∇F needed for evaluating the tensor. The choice of high-order interpolation schemes is justified by the fact that the eigenvector field derived from the Cauchy-Green strain tensor is globally orientable, as demonstrated in Serra and Haller (2017). This global orientability allows for the use of cubic or spline interpolation schemes, which are more accurate than the linear interpolation methods typically required in non-orientable direction fields. This theoretical advantage contributes to more precise and fully automated detection of coherent vortex boundaries. We have revised the text as follows:

- Original sentence:

"Given the three-dimensional nature of the flow field-encompassing longitude, latitude, and time-as particles move, they necessitate interpolation. Striving for an optimal balance between precision and computation speed, we employ the Cubic interpolation method and use B-splines as the mixing matrix. The Cubic interpolation method provides third-order spline interpolation, thereby ensuring high-order continuity of the velocity field post-interpolation and circumventing significant errors induced by numerical sensitivity."

- Revised sentence:

"Given the three-dimensional nature of the flow field-encompassing longitude, latitude, and time-as particles move, interpolation is required to estimate their trajectories. The cubic interpolation and B-splines are applied for specifically in the computation of the Cauchy-Green strain tensor field, which underlies the variational eddy boundary detection. This high-order interpolation ensures smooth and accurate estimation of the deformation gradient ∇F, which is essential for evaluating the tensor. The use of cubic interpolation is justified by the global orientability of the eigenvector field derived from the Cauchy-Green strain tensor, as shown in Serra and Haller (2017). This property enables the use of high-order schemes such as B-splines, which offer superior accuracy and continuity compared to linear interpolation typically used in non-orientable direction fields. This theoretical advantage contributes to the precise and fully automated detection of coherent Lagrangian vortex boundaries." (Please see Line 297-305)

Reference:

Serra, M. and Haller, G.: Efficient computation of null geodesics with applications to coherent vortex detection, Proceedings of the Royal Society a-Mathematical Physical and Engineering Sciences, 473, https://doi.org/10.1098/rspa.2016.0807, 2017.

③**Response:** Thank you for pointing this out. We have added detailed algorithmic pseudocode to Appendix A, which outlines the numerical procedure used to extract BHEs via null geodesic calculation. The pseudocode explicitly includes the main steps performed by the PYTHON code with GPU parallelization. This addition aims to make the implementation transparent and reproducible for users. we have added the pseudocode as follows: (Please see Line 751-754)

Appendix A: GPU-Accelerated Pseudo-Code for BHE Detection Using Null Geodesics

Algorithm 1 provides a brief summary of the main steps performed by the PYTHON code for BHE detection based the GPU-accelerated null geodesics algorithm.

Algorithm 1. BHE detection based the GPU-accelerated null geodesics algorithm.

Input:

- u(x, y, t), v(x, y, t): 2D velocity field in time window $[t_0, t_1]$
- Range of stretch rate $\lambda \in [0.9, 1.1]$ with step size $\Delta\lambda$
- Grid of initial polar angle $\theta \in [0, 2\pi]$
- dt: integration time step
- grid resolution: Resolution of the grid
- domain bounds: Domain boundaries for the particle seeding

#Step 1: Load preprocessed velocity field (u, v)

```
VelocityField,velocity_field = load_velocity_data("u_field.nc", "v_field.nc");
```

#Step 2: Initialize particle grid and allocate on GPU(see equations (5) and (6))

```
Grid grid = initialize_uniform_grid(resolution=1/32.0);
Particle* d_particles = allocate_particles_on_GPU(grid);
```

#Step 3: Integrate particle trajectories using RK4 (GPU parallelism)

```
RK4_integrate<<<gridDim, blockDim>>>(velocity_field, d_particles, time_step=0.1, total_steps);
```

#Step 4: Compute Cauchy-Green strain tensor on GPU (see equation (3))

```
compute_CG_tensor<<<gridDim, blockDim>>>(d_particles, d_C_tensor, grid);
```

#Step 5: Perform eigendecomposition for each C tensor (on GPU) (see equation (4))

```
eigendecompose_C_tensor<<<gridDim, blockDim>>>(d_C_tensor, d_lambda1, d_lambda2, d_xi1, d_xi2);
```

#Step 6: Extract null geodesics using ZeroSet condition (each thread for φ at fixed λ) (see equation (12))

```
for λ in linspace (0.9, 1.1, 9):    #λ from 0.9 to 1.1 with step of 0.025
    extract_null_geodesics<<<gridDim_lambda,blockDim_phi>>>(d_C_tensor,λ,d_phi_array,d_null_flags);
```

#Step 7: Check which geodesics are closed based on φ = 2π

```
check_geodesic_closure<<<gridDim_geo, blockDim>>>(d_null_flags, d_phi_diff_array, d_closed_flags);
```

#Step 8: Integrate valid seed points inside tensor field to generate nested LCS(see equations (13) and (14))

```
integrate_seeds_in_tensor<<<gridDim_seed,blockDim>>>(d_C_tensor,d_seeds,d_nested_curves,target_phi=2π);
```

#Step 9: Transfer nested curves to CPU for clustering and BHE boundary extraction

```
download_curves_to_host(d_nested_curves, nested_curves);
clusters = cluster_curves_by_centroids(nested_curves);
for cluster in clusters:
    areas = compute_area_of_each_curve(cluster);
    max_curve = find_curve_with_max_area(areas);
    save_BHE_boundary(max_curve);
```

Output:

- A closed contour representing the boundary of Black Hole Eddy

In addition, we have already included performance benchmarks for the GPU-based algorithm in the Performance Analysis section (Section 2.2.3), where we quantitatively demonstrate the significant computational speedup over the CPU implementation. These benchmarks go beyond the January 1, 2021 test case and provide a broader evaluation of the algorithm's efficiency.

We hope these additions address your concerns regarding both the algorithmic detail and performance evaluation.

Comment:

2. No Quantitative Validation of Detected Eddies

The manuscript claims that BHEs are materially coherent and more robust than Eulerian or RCLV eddies, *but provides no external validation:*

① There is no comparison with drifter trajectories, synthetic benchmark fields, or any ground-truth datasets.

② The evaluation is purely qualitative, relying on particle advection visualizations (Figures 8–9) without metrics like retention rate, edge dispersion, or FTLE gradients.

③ The label "Naked BHEs" is introduced based on non-overlap with other eddy datasets, but without a demonstration that these features correspond to real or meaningful oceanic structures.

①Response: We appreciate your suggestion regarding validation. To address this point, we have incorporated a comparison with the recently published Rotationally Coherent Lagrangian Vortices (RCLVs) dataset by Liu and Abernathey (2022), which serves as a well-established reference for materially coherent eddy detection in the North Pacific. The boundaries of RCLVs are shown as black contours in Fig. 8, overlaid with our identified BHEs (red for anticyclonic, blue for cyclonic).

Moreover, to provide a more physically meaningful background and enhance the interpretability of eddy structures, we have computed the Finite-Time Lyapunov Exponent (FTLE) field based on the same Lagrangian particle advection used for BHE detection. The FTLE field (displayed as the background color in Fig. 8) highlights regions of strong stretching and deformation, which generally align with eddy boundaries. The use of FTLE as a Lagrangian diagnostic further supports the coherence of the identified BHEs and provides a robust frame for comparison with existing vortex datasets.

These additions strengthen the validation of our method and demonstrate the consistency of our results with previously published Lagrangian eddy datasets. We have added the paragraph as follows:

"To validate the identified BHEs, we conducted a spatial comparison with the RCLVs dataset published by Liu and Abernathey (2022). As shown in Fig. 8a, the spatial distribution of BHEs (outlined in red for anticyclonic and blue for cyclonic) is overlaid on the FTLE (Finite-Time Lyapunov Exponent) field, with RCLVs marked by black contours. The FTLE background highlights regions of strong Lagrangian stretching, which often correspond to the material boundaries of coherent structures. Two representative regions are enlarged in Fig. 8b and Fig. 8c, in both regions, BHE boundaries closely align with high FTLE ridges and frequently overlap with or are enclosed within RCLV, indicating that the identified BHEs represent materially coherent eddy structures." (Please see Line 454-465)

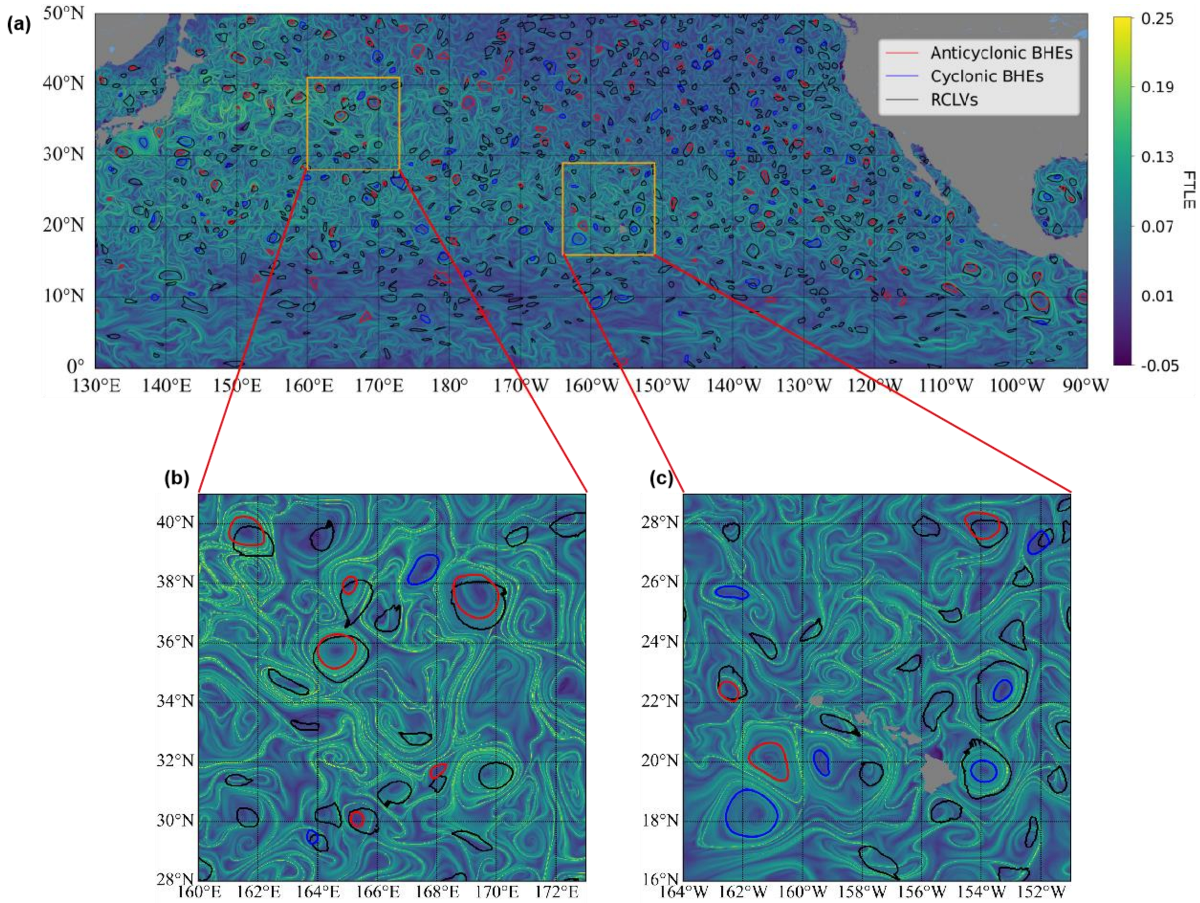

Figure 8. Comparison between identified Black Hole Eddies (BHEs) and Rotationally Coherent Lagrangian Vortices (RCLVs) from Liu et al. (2022) over the North Pacific on 1 March 2019, over a detection time interval T = 30 days with the Finite-Time Lyapunov Exponent (FTLE) field superimposed. (a) Anticyclonic BHEs are outlined in red, cyclonic BHEs in blue, and RCLVs in black. (b, c) Zoomed-in views of two representative regions.

Reference:

Liu, T. and Abernathey, R.: A global Lagrangian eddy dataset based on satellite altimetry (GLED v1.0), https://doi.org/10.5281/zenodo.7349753, 2022.

②**Response:** We appreciate your insightful comment. We acknowledge that our current description of coherence comparisons via particle trajectories is primarily qualitative and lacks a direct quantitative metric, such as retention rate, edge dispersion, or FTLE gradients. This is a limitation of our current analysis. However, we note that visual inspection of particle trajectories has been widely used in prior studies as a practical and intuitive method to assess the material coherence of eddy boundaries (Haller and Beron-Vera, 2013; Beron-Vera et al., 2015; Xia et al., 2022). In particular, long-term coherence of trapped particles, as demonstrated through their tight confinement within the evolving eddy boundary, has served as an important diagnostic in Lagrangian frameworks. We agree that incorporating quantitative metrics would strengthen the coherence assessment and offer more objective comparisons. We will consider including such measures in future work to complement and validate the visual assessments presented in this study.

Reference:

Haller, G. and Beron-Vera, F. J.: Coherent Lagrangian vortices: the black holes of turbulence, Journal of Fluid Mechanics, 731, https://doi.org/10.1017/jfm.2013.391, 2013.

Beron-Vera, F. J., Wang, Y., Olascoaga, M. J., Goni, G. J., and Haller, G.: Objective Detection of Oceanic Eddies and the Agulhas Leakage, Journal of Physical Oceanography, 43, 1426-1438, https://doi.org/10.1175/JPO-D-12-0171.1, 2013.

Xia, Q., Li, G., and Dong, C.: Global Oceanic Mass Transport by Coherent Eddies, Journal of Physical Oceanography, 52, https://doi.org/10.1175/JPO-D-21-0103, 2022.

③**Response:** Thank you for this insightful comment. In our manuscript, "Naked BHEs" are defined as coherent material eddies that satisfy the definition of BHEs, but do not overlap with eddies identified by traditional Eulerian and RCLVs methods. While these eddies may not exhibit strong vorticity signatures, they do demonstrate high material coherence over time, as confirmed by Lagrangian analysis.

We agree that further validation is necessary to establish whether these features correspond to physically meaningful oceanic structures. In the current work, our goal is to highlight the existence of such coherent features, which may be overlooked by conventional detection techniques. We consider this a preliminary but important step toward understanding a broader spectrum of oceanic transport phenomena. We have clarified this point in the conclusions section and intend to investigate the physical nature and dynamical relevance of these "Naked BHEs" in future work. The newly added sentence is:

"However, the correspondence between NBHEs and physically meaningful oceanic structures remains unclear and further investigation is needed to understand their underlying dynamics." (Please see Line 702-703)

Comment:

3. Unsubstantiated Claims about Transport Accuracy

The authors state that BHEs provide a more accurate estimate of oceanic transport (e.g., 1.5 Sv westward flow), but this assertion is flawed:

① "More accurate" implies a benchmark or truth reference, which is not provided. It is inappropriate to *draw such conclusions by comparison to RCLV fluxes alone.*

② *The transport calculation uses idealized assumptions: a fixed eddy depth (500 m), spherical geometry, and a simplified volume formulation V=sπR^2h. The sensitivity of transport estimates to these assumptions is never tested.*

③ *Only zonal and meridional surface-integrated transports are estimated; vertical fluxes, submesoscale exchange, and isopycnal pathways are ignored despite their relevance to the stated motivation (e.g., biogeochemical cycles).*

The transport interpretation should be reframed more cautiously, or supplemented with uncertainty ranges and alternate metrics (e.g., Lagrangian coherence indicators).

①**Response:** We appreciate your insightful comment. We have revised the sentence to avoid suggesting a definitive accuracy benchmark. The new phrasing emphasizes the relatively small transport by BHEs and interprets this result in the broader context of mesoscale dynamics, highlighting that strongly coherent eddy cores may only account for a limited fraction of total transport. This interpretation is consistent with previous studies suggesting that incoherent processes may dominate lateral material exchange (Hausmann and Czaja, 2012; Abernathey and Haller, 2018). We have revised the text as follows:

- Original sentence:

"This consistency in transport allows for more accurate assessments of material transport capacity compared to the overestimated values often associated with RCLVs."

- Revised sentence:

"While the transport by BHEs is relatively small (~1.5 Sv), it does not imply insignificance. Instead, it suggests that strongly coherent eddy cores may contribute only a minor fraction to the total mesoscale transport, which contrasts with the potentially overestimated values associated with RCLVs. Our findings align with the view that the dominant contribution to mesoscale transport may arise not from coherent eddy cores, but from incoherent processes such as stirring and filamentation at the periphery (Hausmann and Czaja, 2012; Abernathey and Haller, 2018)." (Please see Line 725-730)

Reference:

Abernathey, R. and Haller, G.: Transport by Lagrangian Vortices in the Eastern Pacific, Journal of Physical Oceanography, 48, 667-685, https://doi.org/10.1175/JPO-D-17-0102.1, 2018.

Hausmann, U. and Czaja, A.: The observed signature of mesoscale eddies in sea surface temperature and the associated heat transport, Deep-Sea Res. Pt. I, 70, 60-72, https://doi.org/10.1016/j.dsr.2012.08.005, 2012.

②**Response:** Thank you for your comment. We agree that our transport calculation is based on simplified assumptions, including a fixed eddy depth of 500 m, spherical geometry, and a volume formulation. These assumptions follow common practice in mesoscale eddy studies. Specifically, the eddy depth of 500 m is based on observational evidence from Zhang et al. (2014), who show that warm-core eddies typically trap fluid with closed PV contours down to this depth. The shape correction coefficient s=0.5 is adopted following Dong et al. (2014), who suggest this as a conservative estimate accounting for the bowl-like geometry of eddies. The main focus of this paper is to compare the eddy-induced volume transport by BHEs and RCLVs based statistical means. Our results show that the transport by BHEs is approximately one-third of that by RCLVs, which is consistent with previous findings (Dong et al., 2012). Our objective in estimating eddy transport is not to determine the absolute magnitude with high precision, but rather to enable a consistent relative comparison between BHEs and the reference RCLVs dataset. As both estimates adopt the same assumptions and formulation, the relative differences are robust and meaningful within the context of this study.

Reference:

Dong, C. M., McWilliams, J. C., Liu, Y., and Chen, D. K.: Global heat and salt transports by eddy movement, Nature Communications, 5, https://doi.org/10.1038/ncomms4294, 2014.

Zhang, Z. G., Wang, W., and Qiu, B.: Oceanic mass transport by mesoscale eddies, Science, 345, 322-324, https://doi.org/10.1126/science.1252418, 2014.

③**Response:** We appreciate your comment and agree that this is a limitation of the present study. We have added sentences in the conclusions section to explicitly acknowledge this constraint and to clarify that future work will aim to incorporate vertical and submesoscale fluxes, as well as isopycnal transport pathways, to advance our understanding of eddy-mediated transport in a more comprehensive framework. The newly added sentence is:

"However, the estimates are limited to zonal and meridional surface-integrated transports and are constrained by the use of idealized assumptions in the transport calculation. Vertical fluxes, submesoscale exchanges, and isopycnal pathways-which are particularly relevant for understanding biogeochemical cycles-are not considered in this study. More attention is required to incorporate these additional processes to provide a more complete picture of eddy-driven material transport." (Please see Line 730-733)

Comment:

4. Sensitivity to Detection Parameters Not Assessed

① The results are strongly conditioned on two key detection parameters:

- *Minimum eddy radius: 20 km*
- *Minimum lifespan: 4 weeks*

Response: We appreciate your thoughtful comment regarding the potential sensitivity of our results to detection parameters. While a full sensitivity analysis is beyond the scope of this study, we acknowledge the importance of this point. However, we would like to clarify that our choice of a minimum eddy radius of 20 km is grounded in established practices in previous studies.

Specifically, this radius threshold corresponds to approximately 4 pixels in the 0.25° gridded AVISO ADT dataset, which aligns with the minimal resolvable scale based on the spatial characteristics of the AVISO product. Specifically, Faghmous et al. (2015) applied a 4-pixel minimum size threshold when identifying eddies in AVISO SLA data with 0.25° resolution, which corresponds approximately to a radius of 20 km. This criterion has been widely adopted in the literature to ensure that detected features are adequately resolved while avoiding the inclusion of noise and subgrid-scale artifacts.

Regarding the minimum eddy lifespan threshold (4 weeks), we acknowledge your concern and appreciate the opportunity to clarify. This threshold is commonly adopted in the mesoscale eddy literature to ensure that only physically meaningful and dynamically coherent eddies are retained.

Short-lived eddies often reflect transient noise, poorly defined structures, or tracking artifacts. As such, previous studies have frequently excluded these to enhance the robustness of statistical and dynamical analyses, while longer-lived eddies contribute more significantly to transport and ecological processes (Chelton et al., 2011b; Faghmous et al., 2015; Chen and Han, 2019). Additionally, recent Lagrangian-based global eddy dataset (Liu and Abernathey, 2023) has applied 30-day lifetime constraints to filter out unstable or ambiguous features, which is similar to our study.

To make this clearer, we have added a brief explanation of this parameter choice in Section 3.1 of the revised manuscript.

"The minimum radius threshold of 20 km (approximately 4 pixels at the 0.25° resolution of the ADT data) is applied to ensure the physical relevance and adequate resolution of detected eddies, consistent with the filtering approach of Faghmous et al. (2015) to exclude unresolved or noisy features. Additionally, the minimum lifetime of 4 weeks (28 days) is imposed to retain only coherent and dynamically meaningful structures, following common practices in mesoscale eddy studies (Chelton et al., 2011b; Faghmous et al., 2015; Chen and Han, 2019; Liu and Abernathey, 2023)." (Please see Line 467-471)

Reference:

Chelton, D. B., Schlax, M. G., and Samelson, R. M.: Global observations of nonlinear mesoscale eddies, Progress in Oceanography, 91, 167-216, https://doi.org/10.1016/j.pocean.2011.01.002, 2011b.

Chen, G., and Han, G.: Contrasting short-lived with long-lived mesoscale eddies in the global ocean, Journal of Geophysical Research: Oceans, 124, 3149–3167, https://doi.org/10.1029/2019JC014983, 2019.

Faghmous, J. H., Frenger, I., Yao, Y. S., Warmka, R., Lindell, A., and Kumar, V.: A daily global mesoscale ocean eddy dataset from satellite altimetry, Scientific Data, 2, https://doi.org/10.1038/sdata.2015.28, 2015.

Liu, T. Y. and Abernathey, R.: A global Lagrangian eddy dataset based on satellite altimetry, Earth System Science Data, 15, 1765-1778, https://doi.org/10.5194/essd-15-1765-2023, 2023.

However, there is no sensitivity analysis to show how the number, distribution, or transport of BHEs would change with different thresholds.

- *For instance, a minimum radius of 20 km excludes coastal or tropical eddies that may be materially coherent.*
- *A stricter lifespan cutoff (>6 or 8 weeks) may reveal whether BHEs are truly more persistent than RCLVs.*
- *Parameter tuning can significantly impact regional eddy statistics, yet this issue is not acknowledged or tested.*

Without such analysis, the robustness and generalizability of the dataset remain in question.

Response: We appreciate your insightful comment regarding the sensitivity of the detection results to parameter choices. We acknowledge that the choice of a 20 km minimum radius threshold, while motivated by the need to ensure feature detectability and coherence within the resolution of the SLA dataset, may exclude smaller-scale eddies, particularly those in coastal or tropical regions, that are nonetheless materially coherent. This trade-off between resolution-limited detection and inclusiveness is a common constraint in mesoscale eddy identification based on altimetry-derived fields. Future efforts could incorporate multi-scale detection approaches or higher-resolution datasets to better capture submesoscale or marginally resolved features. While we agree that stricter lifespan thresholds (e.g., 6-8 weeks) could offer insight into the persistence of BHEs relative to RCLVs, we chose 4 weeks to strike a balance between data availability, computational feasibility, and statistical representativeness. Furthermore, the same radius and lifespan thresholds were applied when detecting RCLVs for intercomparison, ensuring consistency in the evaluation of both eddy types. We fully acknowledge that detection parameters can affect eddy statistics. While a comprehensive sensitivity analysis is beyond the scope of the present study, we now explicitly acknowledge this limitation and the sensitivity analysis of Black Hole Eddies statistics under different parameter regimes to be explored in future work. The newly added sentence is:

"While these threshold values are commonly adopted and physically justified, regional eddy statistics may be sensitive to variations in these parameters." (Please see Line 475-476)

"And further investigations involving systematic sensitivity tests would help assess the robustness of the results under alternative detection criteria." (Please see Line 746-747)

Comment:

5. Geographic Scope Is Narrow but Conclusions Overreach

While the dataset is limited to the North Pacific (0-50°N, 130-270°E), the authors occasionally use language implying broader relevance (e.g., "first BHE dataset", "offers better transport estimates").

However:

• The dynamical regimes of the Southern Ocean, Atlantic, or Indian Ocean differ considerably (e.g., stronger stratification, different baroclinic instability characteristics).

• It is unclear whether the same GPU algorithm, with its current assumptions and thresholds, would perform well in those settings.

The manuscript should clearly delimit the spatial scope of its claims and avoid generalizations unless supported by multi-basin tests or transferability arguments.

Response: We appreciate your insightful comment and agree that the current study's conclusions should be interpreted within the geographic context of the North Pacific. In response, we have carefully revised the manuscript to more clearly delimit the spatial scope of our findings. Specifically, we now explicitly state that our dataset and results pertain solely to the North Pacific region (0-50°N, 130-270°E) and that any broader implications are tentative and subject to further validation. We have also removed or softened language that previously implied global generality. Additionally, we now acknowledge in the conclusions section that dynamical regimes and detection challenges may differ in other ocean basins, and that the applicability of our GPU-based method to those regions remains to be evaluated in future work. We have revised the text as follows:

- Original sentence:

  "This study presents an efficient Graphics Processing Unit (GPU) -based BHE identification algorithm, enhancing computational efficiency by approximately 13 times compared to the existing methods. Using this algorithm, the North Pacific a Black Hole Eddy dataset (BHE v1.0) is constructed for the first time, based on satellite-derived surface geostrophic velocity data from January 1, 1993 to May 5, 2023 (Tian, F. L., Zhao, Y. Y., Long, S., and Chen, G.: A Black Hole Eddy Dataset of North Pacific Ocean Based on Satellite Altimetry (BHE v1.0). Zenodo [dataset], https://doi.org/10.5281/zenodo.15597447, 2025a)."

- Revised sentence:

  "This study presents an efficient Graphics Processing Unit (GPU) -based BHE identification algorithm for the North Pacific (0-50°N, 130°-270°E), enhancing computational efficiency by approximately 13 times compared to the existing methods. Using this algorithm, a Black Hole Eddy dataset (BHE v1.0) is constructed for the first time in the North Pacific Ocean, based on satellite-derived surface geostrophic velocity data from January 1, 1993 to May 5, 2023 (Tian et al., 2025a)." (Please see Line 13-16)

- Original sentence:

  "As shown in Fig. 16, both eddy types contribute predominantly westward (negative) zonal transport across the North Pacific basin. However, BHEs exhibits notably weaker westward transport intensity compared to RCLVs, particularly between 10° N and 30°N (Fig. 18a and Fig. 18c)."

- Revised sentence:

  "Within the North Pacific, as shown in Fig. 20, both eddy types contribute predominantly westward (negative) zonal transport across the North Pacific basin. However, BHEs exhibits notably weaker westward transport intensity compared to RCLVs, particularly between 10° N and 30°N (Fig. 20a and Fig. 20c)". (Please see Line 628)

- Added sentence:

  "It is important to note that this study focuses exclusively on the North Pacific (0-50°N, 130°E-270°E), and all algorithm validation and dataset construction are limited to this region, due to its high eddy activity (Chelton et al., 2011b), extensive altimetric coverage favorable for coherent eddy detection (Chaigneau et al., 2008)." (Please see Line 739-741)

Minor Comments:

While the authors appear to have used AI tools to address language issues-a practice I generally view with caution-I appreciate their transparency in doing so. That said, this manuscript still contains some grammar issues, vague phrasing, a lot of missing space inbetween, redundancy, missing or imprecise terminology, and unclear logic in comparisons, and many are obviously related to the use of AI tools. Many technical points are introduced without context or explained in overly dense mathematical language without intuitive guidance. Frankly, if AI-generated text plays a significant role in scientific writing, it raises serious questions about why human reviewers should invest their time and expertise in carefully reading and critiquing such work. While I understand the appeal of these tools, I find their use in this context concerning and, in principle, disagreeable. I believe this issue merits ongoing discussion within the scientific community.

Following I list some, but not all issues with the writing. The authors should seriously consider taking a more thorough proofreading process or seeking assistance from a native English speaker, rather than relying heavily on AI tools for language editing. I expect to see substantial improvement than revising the listed issues.

Response: We appreciate your thoughtful comments and concerns regarding the use of AI tools in academic writing. We would like to clarify that AI-based tools were only used to assist with surface-level English polishing, such as grammar, spelling, and sentence fluency. At no point did we use AI tools to generate technical content, introduce domain-specific terminology, or compose the scientific arguments in the manuscript. All domain knowledge, analyses, and interpretations are entirely based on the authors' understanding and expertise.

Following the suggestions, we have carefully revised the manuscript to address the specific language issues that were listed, including grammar corrections, reducing redundancy, clarifying vague or imprecise terminology. In addition, we performed a thorough re-read of the entire manuscript and made further improvements beyond the reviewer's list, including enhancing transitions, simplifying complex sentences, and rephrasing several technical explanations to be more intuitive and accessible.

We fully understand and respect the reviewer's concerns about the broader implications of AI usage in scientific writing. We share the view that AI tools should be used responsibly and transparently, and only as a support mechanism. We hope the revised version meet with your approval.

Thank you again for your valuable feedback, which significantly helped us improve the manuscript.

Comment 1: *Line 10: "The methodologies employed for the identification of ocean coherent eddies..."*

"ocean coherent eddies" should be "coherent ocean eddies".

Response: Thank you for your helpful comment. "ocean coherent eddies" was reworded as "coherent ocean eddies". (Please see Line 10)

Comment 2: *Line 21: "maintain strong material coherence and are able to maintain concentration..."*

Repetitive "maintain"; also "concentration" is ambiguous here. Better: "retain material coherence throughout their lifecycle without significant mixing."

Response: Thank you for pointing this out. We agree that the original sentence was repetitive and the term "concentration" could be ambiguous. We have revised the sentence to: "... retain material coherence throughout their lifecycle without significant mixing." This revision improves clarity and avoids redundancy. (Please see Line 21-22)

Comment 3: *Line 24: "Transport analysis shows that BHEs induce westward transport about 1.5 Sv…"*

Missing "of": should be "transport of about 1.5 Sv".

Response: Thank you for your careful reading. We have corrected the sentence by adding the missing preposition, and it now reads: "Transport analysis shows that BHEs induce westward transport of about 1.5 Sv…" (Please see Line 26)

Comment 4: *Line 26: "These finding addresses…"*

Should be "These findings address…"

Response: Thank you for pointing out the grammatical error. We have corrected the sentence to: "These findings address…" in the revised manuscript. (Please see Line 27)

Comment 5: *Line 30: "Mesoscale eddies…can persist for periods of a few weeks to several years." "Periods of a few weeks" is vague; could benefit from citing eddy lifetimes more precisely by region.*

Response: We thank the reviewer for this helpful suggestion. We agree that "a few weeks to several years" is vague and that it is more informative to provide region-specific statistics. Since our study focuses on the North Pacific region (130°E-270°E, 0-50°N), we have revised the sentence to reflect typical eddy lifetimes in this area. Based on Chelton et al. (2011a) and Chen and Han (2019), most mesoscale eddies in the North Pacific have lifetimes ranging from approximately 4 weeks to one year, with long-lived eddies-persisting more than 12 months-being less frequent but still significant, especially in regions influenced by strong currents such as the Kuroshio Extension. We have revised the text as follows:

- Original sentence:

"Mesoscale eddies, one of the most prominent processes in the ocean, typically range from several tens to hundreds of kilometers in spatial scale and can persist for periods of a few weeks to several years (Chelton et al., 2011a; Chaigneau et al., 2008)"

- Revised sentence:

"Mesoscale eddies, one of the most prominent processes in the ocean, typically range from several tens to hundreds of kilometers in spatial scale (Chelton et al., 2011a; Chaigneau et al., 2008). In the North Pacific, mesoscale eddies generally persist for about 4 weeks to one year, with longer-lived structures occasionally exceeding this range, particularly in energetic regions such as the Kuroshio Extension (Chelton et al., 2011a; Chen and Han, 2019)." (Please see Line 31-35)

Reference:

Chaigneau, A., Gizolme, A., and Grados, C.: Mesoscale eddies off Peru in altimeter records: Identification algorithms and eddy spatio-temporal patterns, Progress in Oceanography, 79, 106-119, https://doi.org/10.1016/j.pocean.2008.10.013, 2008.

Chelton, D. B., Gaube, P., Schlax, M. G., Early, J. J., and Samelson, R. M.: The Influence of Nonlinear Mesoscale Eddies on Near-Surface Oceanic Chlorophyll, Science, 334, 328-332, https://doi.org/10.1126/science.1208897, 2011a.

Chen, G., and Han, G. Y.: Contrasting short-lived with long-lived mesoscale eddies in the global ocean, Journal of Geophysical Research: Oceans, 124, https://doi.org/10.1029/2019JC014983, 2019.

Comment 6: *Line 34: "influencing on the marine material cycle…"*

Remove "on". Correct: "influencing the marine material cycle…"

Response: Thank you for your suggestion. We have removed the unnecessary preposition, and the sentence now reads: "influencing the marine material cycle…" in the revised manuscript. (Please see Line 36)

Comment 7: *Line 36: "Eulerian approachs…" Should be "approaches".*

Response: Thank you for pointing out the typo. We have corrected "approachs" to "approaches" in the revised manuscript. (Please see Line 40)

Comment 8: *Lines 41–45: There is repetition about Eulerian methods being "frame-dependent" and generating filamentation. This point is important but can be made more concisely.*

Response: Thank you for your insightful comment. We agree that the original paragraph was somewhat repetitive in describing the limitations of Eulerian methods, particularly regarding frame-dependence and filamentation. To improve clarity and conciseness, we have revised the text as follows:

- Original sentence:

"However, the Eulerian methods have several limitations. On the one hand, they are frame-dependent, while the flow field in the real ocean does not have a fixed reference frame, which may cause the results to lose coherence under coordinate transformation. On the other hand, due to the transient characteristics of the fluid, a body of water within the Eulerian boundary can generate numerous filaments and disperse rapidly, potentially resulting in an overestimation of the eddy's transport capacity."

- Revised sentence:

"However, Eulerian methods are frame-dependent and may yield inconsistent results under coordinate transformations, as the ocean flow lacks a fixed reference frame. Moreover, due to the transient nature of ocean currents, water parcels within Eulerian-defined boundaries can quickly stretch into filaments and disperse, leading to a possible overestimation of eddy transport." (Please see Line 45-48)

Comment 9: *Line 49: "provide a more accurate structures…"*

Should be: "provide more accurate representations of structures…"

Response: Thank you for your suggestion. We have revised the sentence to: "provide more accurate representations of structures…" to improve clarity and grammatical accuracy. (Please see Line 57)

Comment 10: *Line 52: "Before Lagrangian coherent structures (LCSs) entered into strict mathematical definitions…"*

Wordy. Consider simplifying to "Before LCSs were rigorously defined…"

Response: We appreciate your careful attention to language clarity throughout the manuscript. We have revised the sentence to: "Before LCSs were rigorously defined…" to make it more concise and readable. (Please see Line 60)

Comment 11: *Lines 62–63: "These eddies boundary are commonly referred to as the 'photon sphere, analogous to Black Holes…"*

"Eddies boundary" should be "eddy boundaries"; quotation wrong; analogy should be better justified.

Response: Thank you for pointing this out. We have revised the text to more accurately describe the analogy with black holes. Specifically, we clarified that the mathematical formulation of the Lagrangian vortex boundaries introduced by Haller and Beron-Vera (2013) is equivalent to the definition of photon spheres in general relativity. We also elaborated on the physical interpretation, noting that particles crossing the vortex boundary become trapped, thereby justifying the term "Black Hole Eddy." We have revised the text as follows:

- Original sentence:

"Subsequently, Haller and Beron-Vera (2013) introduced a rigorous variational framework for detecting coherent Lagrangian vortices. These vortices are defined as closed material curves that exhibit minimal stretching and filamentation throughout their evolution, providing an objective basis for identifying coherent structures in unsteady flows. These eddy boundaries are commonly referred to as the "photon sphere, analogous to Black Holes found in the universe". Once matter crosses this boundary, it becomes trapped within the eddy, there giving rise to the term Black Hole Eddy (BHE)."

- Revised sentence:

"Subsequently, Haller and Beron-Vera (2013) introduced a rigorous variational framework for detecting coherent Lagrangian vortices, which are defined as closed material loops that remain minimally stretched and filamented over time. Remarkably, the mathematical formulation of these vortex boundaries is analogous to that of photon spheres in general relativity. In this analogy, fluid particles that cross the boundary remain trapped within the eddy's interior, much like light trapped inside a black hole. This striking similarity has led to the term "Black Hole Eddy" (BHE) being used to describe such coherent Lagrangian vortices." (Please see Line 68-73)

Reference:

Haller, G. and Beron-Vera, F. J.: Coherent Lagrangian vortices: the black holes of turbulence, Journal of Fluid Mechanics, 731, https://doi.org/10.1017/jfm.2013.391, 2013.

Comment 12: *Line 164: "...includes daily-averaged, 5-day-averaged, 8-day-averaged…"*

Repetitive and wordy. Consider: "Includes datasets with daily to monthly averages…"

Response: Thank you for the suggestion. We have revised the sentence to: "…includes datasets with daily to monthly averages…" to avoid repetition and improve conciseness. (Please see Line 176)

Comment 13: *Line 214: "...indicative of the BHE."*

The sentence assumes readers understand how minimal deformation implies BHE status—should elaborate or clarify.

Response: We thank the reviewer for pointing out the lack of clarity in this sentence. We agree that the connection between minimal boundary deformation and the definition of a Black Hole Eddy requires further elaboration.

In the variational framework introduced by Haller and Beron-Vera (2013), materially coherent eddies are defined as regions enclosed by closed null-geodesics of a Lorentzian metric derived from the Cauchy-Green strain tensor (See equation (14)). These boundaries resist the typical stretching and folding seen in chaotic flows, thereby undergoing minimal deformation during advection. The absence of filamentation and sustained compactness of the boundary over time indicate strong material coherence. Therefore, boundaries that exhibit such minimal deformation are used to identify Black Hole Eddies. To clarify this point, we have revised the sentence as follows:

- Original sentence:

"Black Hole Eddy, characterized by their boundaries undergoing minimal deformation during advection by the flow field, do not generate noticeable filamentous structures while in motion. The identification of these eddies employs a variational method, recognizing the boundaries as closed null geodesics defined by an appropriate Lorentz metric across the flow domain. The primary challenge lies in accurately pinpointing the boundaries that exhibit minimal deformation, which is indicative of the BHE."

- Revised sentence:

"Black Hole Eddies are characterized by boundaries that undergo minimal deformation during advection, maintaining a compact shape without generating filamentous structures (Haller and Beron-Vera, 2013). This minimal deformation reflects their material coherence and serves as a key criterion for their identification."

(Please see Line 223-226)

Reference:

Haller, G. and Beron-Vera, F. J.: Coherent Lagrangian vortices: the black holes of turbulence, Journal of Fluid Mechanics, 731, https://doi.org/10.1017/jfm.2013.391, 2013.

Comment 14: *Line 224: "...relies on direction field integration…"*

"direction field" could be more accurately termed "vector field" or "eigenvector field (?)

Response: Thank you for the valuable suggestion. We agree that "vector field" is a more accurate term than "direction field" in this context. We have revised the phrase to "relies on vector field integration" in the updated manuscript. (Please see Line 233)

Comment 15: *Line 230: "This paper utilizes CUDA as a multithreaded toolkit…"*

Passive voice preferred in scientific writing: "CUDA was used…"

Response: Thank you for the suggestion. We have revised the sentence to use passive voice as recommended. It now reads: "CUDA was used as a multithreaded toolkit…" in the revised manuscript. (Please see Line 246)

Comment 16: *Line 272: "effectively mitigates the noise in the eigenvector field…"*

Needs clarification of how densification and interpolation specifically mitigate numerical noise.

Response: Thank you for your valuable comment. We agree that the original explanation lacked clarity regarding how auxiliary points help reduce numerical noise in the eigenvector field. To address this, we revised the paragraph to elaborate on the mechanism. Specifically, by adjusting the spacing of auxiliary points (i.e., δ_{x_1} and δ_{x_2}), we can more accurately compute the deformation gradient tensor through finite differences. This reduces discretization error, leading to smoother eigenvector fields and improving robustness against numerical sensitivity in eigenvector direction estimation. We have revised the text as follows:

- Original sentence:

"Specifically, by altering the dimensions of δ_{x_1} and δ_{x_2}, we can modulate the precision of the main grid node mapping's gradient value. This method effectively mitigates the noise in the eigenvector field and addresses the issue of numerical sensitivity to alterations in the eigenvector direction (Serra and Haller, 2017)."

- Revised sentence:

"Specifically, by adjusting the perturbation sizes of δ_{x_1} and δ_{x_2}, we can refine the numerical approximation of gradients. This local densification reduces discretization error in the finite-difference calculation of the Cauchy-Green strain tensor, thereby smoothing the eigenvector field and alleviating spurious fluctuations caused by numerical sensitivity to eigenvector orientation (Serra and Haller, 2017)." (Please see Line 291-294)

Comment 17: *Line 290: "the program consolidates the output data and transmits it to the host…"*

Wordy. Consider: "Results are compiled and transferred to the host CPU after each step."

Response: Thank you for the helpful suggestion. We have revised the sentence to: "results are compiled and transferred to the host CPU after each step." to improve clarity and conciseness. (Please see Line 322)

Comment 18: *Line 409-416:*

Non-scientific formatting. Use consistent table structure.

Response: Thank you for your suggestion. We agree that the original formatting of the variable descriptions lacked consistency and scientific clarity. We have reformatted this section into a structured table with uniform field names and detailed descriptions to enhance readability and professionalism. A standardized table listing the variable names, descriptions, and units was added. (Please see Line 451-453)

Field Name	**Description**
key	The name of the BHE.
Eddy ID	Index assigned to each eddy on a specific date, ordered ascendingly from 0.
Eddy Centroid	Longitude (°E) and latitude (°N) of the eddy center, recorded every 7 days.
lon, lat	Longitude and latitude coordinates outlining the boundary of the BHE.
Eddy Area	The area of the BHE, typically in square kilometers.
Eddy Types	Polarity of the BHE, classified as either *Cyclonic* or *Anticyclonic*.

Field Name	Description
Eddy Radius	The radius of the BHE, typically in kilometers.
Lam	The stretch rate associated with each BHE.

Table 1. Description of variables in the BHE dataset in the North Pacific Ocean.

Comment 18: *Line 427: "...anticyclonic\cyclonic eddy..."*

Use proper slash spacing and formatting: "anticyclonic/cyclonic eddy".

Response: Thank you for pointing out the formatting issue. We have corrected the expression to "anticyclonic/cyclonic eddy" with proper slash spacing in the revised manuscript. (Please see Line 481-485)

Comment 19: *Lines 430–450:*

The description of coherence comparisons via particle trajectories is visual but lacks a quantitative metric (e.g., relative dispersion, FTLE, boundary leakage).

Response:

We appreciate the reviewer's insightful comment. We acknowledge that our current description of coherence comparisons via particle trajectories is primarily qualitative and lacks a direct quantitative metric, such as relative dispersion, FTLE, or boundary leakage. This is a limitation of our current analysis. However, we note that visual inspection of particle trajectories has been widely used in prior studies as a practical and intuitive method to assess the material coherence of eddy boundaries (Haller and Beron-Vera, 2013; Wang et al., 201). In particular, long-term coherence of trapped particles, as demonstrated through their tight confinement within the evolving eddy boundary, has served as an important diagnostic in Lagrangian frameworks.

That said, we agree that incorporating quantitative metrics would strengthen the coherence assessment and offer more objective comparisons. We will consider including such measures in future work to complement and validate the visual assessments presented in this study.

Reference:

Haller, G. and Beron-Vera, F. J.: Coherent Lagrangian vortices: the black holes of turbulence, Journal of Fluid Mechanics, 731, https://doi.org/10.1017/jfm.2013.391, 2013.

Wang, Y., Beron-Vera, F. J., and Olascoaga, M. J. Coherent water transport across the South Atlantic, Geophysical Research Letters, 42, 4072-4079, https://doi.org/10.1002/2015GL064089, 2015.

Comment 20: *Line 455:*

"The simultaneous presence of the three eddy types…"

Clarity: Consider rephrasing like "Instances where BHEs overlap both Eulerian eddies and RCLVs are dominated by cyclonic events…"

Response: Thank you for the constructive suggestion. We have rephrased the sentence to improve clarity. The revised version now reads: "Instances where BHEs overlap both Eulerian eddies and RCLVs are dominated by cyclonic events…" (Please see Line 510-511)

Comment 21: *Line 460: "the eddy in Figure 11a and Figure 12a is not initially located around a closed SLA contour or LAVD contour…"*

The sentence is long and ambiguous. Break into two parts: one observation, one interpretation.

Response: Thank you for the helpful suggestion. We agree that the original sentence was overly long and mixed observation with interpretation. We have revised the sentence by splitting it into two parts to improve clarity: one stating the observation, and the other offering a brief interpretation. We have revised the text as follows:

- Original sentence:

"An interesting phenomenon is that the eddy in Figure 11a and Figure 12a is not initially located around a closed SLA contour or LAVD contour, but a coherent structure does exist."

- Revised sentence:

"An interesting phenomenon is observed in Fig. 13a and Fig. 14a: the eddy is not initially located around a closed SLA or LAVD contour. Nevertheless, a coherent structure is still present, suggesting that traditional Eulerian or and RCLVs boundaries may not always capture the full extent of coherent transport." (Please see Line 516-518)

Comment 22: *Line 485: "eddies generated at low latitudes (below 10°N) are very few…"*

Better phrased as "are relatively rare" or "are infrequent".

Response: Thank you for the helpful wording suggestion. We have revised the sentence to: "eddies generated at low latitudes (below 10°N) are relatively rare" for improved readability. (Please see Line 544)

Comment 23: *Line 496: "strongly anticyclonic (red) in the region north of 30°N…"*

Correlation to Kuroshio shear is speculative without supporting reference or cross-analysis.

Response: Thank you for your helpful comment. We acknowledge that the original sentence suggested a causal relationship between the observed polarity distribution and Kuroshio shear without direct supporting evidence. To address this, we have revised the sentence to reflect a more cautious interpretation. While the observed distribution may be influenced by large-scale circulation features and regional dynamics such as baroclinic instability (Chelton et al., 2011b), further analysis would be necessary to clarify the specific role of Kuroshio shear. We have revised the text as follows:

- Original sentence:

"Especially, the polarity distribution of the BHE (Fig. 14a) is strongly anticyclonic (red) in the region north of 30°N, and more cyclonic (blue) between 10°N and 25°N. This distribution pattern is associated with the shear structure of the Kuroshio, baroclinic instability, and other dynamical processes (Chelton et al., 2011b)."

- Revised sentence:

"Especially, the polarity distribution of the BHE (Fig. 16a) is strongly anticyclonic (red) in the region north of 30°N and more cyclonic (blue) between 10°N and 25°N. This distribution pattern may be influenced by large-scale circulation features and regional dynamics such as baroclinic instability (Chelton et al., 2011b)." (Please see Line 555-560)

Reference:

Chelton, D. B., Schlax, M. G., and Samelson, R. M.: Global observations of nonlinear mesoscale eddies, Progress in Oceanography, 91, 167-216, https://doi.org/10.1016/j.pocean.2011.01.002, 2011b.

Comment 24: *Line 549: "we calculate the averaged zonal and meridional transport across the section for each 1° × 1° grid by…"*

Define all symbols clearly; use consistent formatting. Currently hard to parse.

Response: Thank you for your valuable comment. We agree that the description of the calculation on Line 549 was difficult to parse. We have revised this section to clearly define all symbols used in the calculation and have ensured the formatting is consistent for better readability. We have revised the text as follows:

- Original sentence:

"Following the methods used by and Zhang et al. (2014), we calculate the averaged zonal and meridional transport across the section for each 1° × 1° grid by $Q_x = \frac{\sum V \cdot C_x}{N \cdot D_x}, Q_y = \frac{\sum V \cdot C_y}{N \cdot D_y}$, where V, $C_x(C_y)$, and D are the volume, zonal (meridional) propagation speed, and lifetime of an eddy in days, respectively. $\sum$ means the integration of all eddies over the studying period N in days, and $D_x(D_y)$ is the length of one longitude (latitude) degree. The eddy volume is calculated by $V = s\pi R^2 h$, where R is the eddy radius, s = 0.5 is a correction factor for the eddy vertical structure from Dong et al. (2014), and h = 500 m is the eddy depth. Here, BHEs and RCLVs only include eddies that are 28 days are considered from 1993 to 2020."

- Revised sentence:

"Following the methods used by and Zhang et al. (2014), we calculate the averaged zonal and meridional transport across the section for each $1° \times 1°$ grid using the following equations:

$$Q_x = \frac{\sum V \cdot C_x \cdot L}{N \cdot D_x}, Q_y = \frac{\sum V \cdot C_y \cdot L}{N \cdot D_y}$$

Where:

Q_x and Q_y are the average zonal and meridional transports (in m³/s) across the $1° \times 1°$, respectively;

V and L are the volume of each coherent eddy (in m³) and the lifetime of each coherent eddy (in days), respectively;

C_x and C_y are the zonal and meridional propagation speeds (in m/s) of the eddy core;

N is the number of time points (in days) during the detection period;

D_x and D_y is the length of one degree of longitude and latitude (in meters), respectively, at each grid location.

The eddy volume is estimated by the simplified formulation: $V = s\pi R^2 h$, where R is the eddy radius (in meters), $s = 0.5$ is a shape correction factor accounting for the bowl-like vertical structure of eddies, following Dong et al. (2014), and $h = 500m$ is the fixed eddy depth based on the observational analysis by Zhang et al. (2014). In this study, we consider only eddies with lifetimes of at least 28 days, detected between 1993 and 2020, for both BHEs and RCLVs." (Please see Line 610-622)

Reference:

Dong, C. M., McWilliams, J. C., Liu, Y., and Chen, D. K.: Global heat and salt transports by eddy movement, Nature Communications, 5, https://doi.org/10.1038/ncomms4294, 2014.

Zhang, Z. G., Wang, W., and Qiu, B.: Oceanic mass transport by mesoscale eddies, Science, 345, 322-324, https://doi.org/10.1126/science.1252418, 2014.

Comment 25: *Line 560:*

"The peak value…is only about 1.5 Sv, which is three times smaller…"

Prefer "one-third of the RCLV estimate" instead of "three times smaller" (ambiguous and mathematically sloppy).

Response: Thank you for the valuable feedback. We have revised the sentence to: "The peak value…is only about 1.5 Sv, which is one-third of the RCLV estimate" to ensure mathematical clarity and precision. (Please see Line 637)

Comment 26: *Line 580: "…Chen et al. (2021), oceanic eddies have a significant mean egg-like shape…"*

Tone: "Egg-like" may appear informal.

Response: Thank you for the comment regarding the tone of the term "egg-like". We would like to clarify that "egg-like" shape is the formal terminology introduced and used in Chen et al. (2021) to describe the characteristic geometry of oceanic eddies, based on a composite analysis of over 40 million eddy snapshots. Their study reveals that eddies exhibit significant geometric asymmetry and dynamic anisotropy, departing from idealized circular or elliptical assumptions. The term "egg-like" was used to highlight this second-order moment in eddy morphology, which is associated with high-order shape and orientation variability, and is shown to have important implications for ocean dynamics and biological processes.

Given the scientific basis and explicit use of "egg-like" in the referenced peer-reviewed publication, we have retained the term to maintain conceptual consistency and to accurately reflect the anisotropic features emphasized in Chen et al. (2021).

Reference:

Chen, G., Yang, J., and Han, G. Y.: Eddy morphology: Egg-like shape, overall spinning, and oceanographic implications, Remote Sensing of Environment, 257, https://doi.org/10.1016/j.rse.2021.112348, 2021.