Enhancing High-Resolution Forest Stand Mean Height Mapping in China through an Individual Tree-Based Approach with Close-Range LiDAR Data

Dear Editor and Reviewer:

On behalf of my co-authors, we thank you very much for giving us an opportunity to revise our manuscript, and we also appreciate reviewers very much for their positive and constructive comments and suggestions on our manuscript entitled "Enhancing High-Resolution Forest Stand Mean Height Mapping in China through an Individual Tree-Based Approach with Close-Range LiDAR Data" (Manuscript Number: essd-2024-274).

We revised the manuscript according to these comments and suggestions. All changes were marked in highlight text in the revised manuscript. The line numbers in the response are the corresponding line numbers in the revised version.

Once again, thank you very much for your comments and suggestions.

Comment 1:

In the part of Abstract, 'Forest stands mean height is a critical indicator in forestry, playing a pivotal role in various aspects such as forest inventory estimation,' forest inventory estimation is suggested to be modified to forest inventory with various scales, which is more reasonable.

Reply 1: Thank you very much for your professional advice, we have changed 'forest inventory estimation' to 'forest inventory' at Line 21-22.

Comment 2:

In the line of 69: The height metrics from obtained from this approach is forest canopy height, which include not only the actual tree height. There is one mistake in the expression. The sentence should be corrected: The height metrics obtained from this approach is forest canopy height.

Reply 2: The mistake has been corrected according to your kind advices and detailed suggestions. Please refer to Line 69-70 for details.

Comment 3:

In terms of data, various types of data collected over a span of 6 years are included in this manuscript, such as ground measured samples, LiDAR data obtained from different sensors, and remote sensing images. How can these datasets be matched on a temporal scale? Additionally, how can reduce the limitations of images acquired in different years and seasons?

Reply 3: Changes in forest resources tend to occur relatively slowly, and a 5-year period is a sufficiently long-time span to capture significant change trends. The temporal scale for China's national-level forest resource inventory is set at 5 years, aiming to balance the need for real-time data with long-term trend observation. This time span is long enough to detect significant changes in forest ecosystems, yet short enough to ensure that policies and management measures can be promptly adjusted based on the most recent data.

As of 2015, the application of LiDAR has not been widely adopted in forest remote sensing research in China. Considering the cost and the difficulty of data collection, it was challenging to collect extensive, high-point density and accurate data across China within a short timeframe. Considering the nationwide data coverage, the final dataset for this study spans 6 years (one year longer than the time span of the national inventory). This represents a limitation of the data used in this study, which is discussed in the paper. Please refer to Line 485-487 for details.

Comment 4:

The formula of determining coefficients (formula 11), y¯_iis not the mean value for the observed values. y¯ is recommended. In the formula 16, the means of y¯ also should be expressed.

Reply 4: We have corrected the formulas. Please refer to equations 11-20 for details.

Comment 5:

In the manuscript, three accuracy indices were employed to evaluate the performance of models. However, when evaluating results with the same RMSE in various height forests, it is recommended to include rRMSE.

Reply 5: We agreed with the reviewer's comment and added the rRMSE to the Table5, which evaluating results with the same RMSE in various height forests. Please refer to Table 5 for details.

Comment 6:

In Figure 3, it is evident that an overestimation of forest stand height occurs when the weighted average of tree height squared is applied for forest stands taller than 14 meters.

Please provide the underlying reasons.

Reply 6:

We greatly appreciate the reviewer's insightful question. In response, we have explored the issue from both theoretical and empirical perspectives to provide a comprehensive answer. Please refer to Line 440-445 for details.

(1) Theoretical Analysis

Given a set of tree height data $h_1, h_2, \ldots, h_n$ in a plot, and the corresponding diameter at breast height data $d_1, d_2, \ldots, d_n$. Based on the mathematical formulas for h_w and h_L, the following conclusions can be derived.

$$if\ d_i < h_i, then\ h_w < h_L$$
$$if\ d_i \geq h_i, then\ h_w \geq h_L$$

Here is the detailed mathematical proof:

To prove whether the difference between the weighted average heights h_w and h_L, where the weights are $\mathrm{w}_a = h^2$ and $\mathrm{w}_b = d^2$, is greater than or less than zero, we will define and expand the formulas for both weighted averages.

When the diameter at breast height (DBH) d_i is greater than the tree height h_i, $\mathrm{w}_b = d^2 = (h + r)^2$ (with r≥0)

Step1: Define the Weighted Average Heights

Given a set of tree height data $h_1, h_2, \ldots, h_n$, we compute the weighted average heights using weights w_a and w_b as follows:

Weighted average height h_w using weights $\mathrm{w}_a = h^2$:

$$h_w = \frac{\sum_{i=1}^{n} h_i * {h_i}^2}{\sum_{i=1}^{n} {h_i}^2} = \frac{\sum_{i=1}^{n} {h_i}^3}{\sum_{i=1}^{n} {h_i}^2}$$

Weighted average height h_L using weights $\mathrm{w}_b = (h + r)^2$:

$$h_L = \frac{\sum_{i=1}^{n} h_i * (h_i + r)^2}{\sum_{i=1}^{n} (h_i + r)^2}$$

Expand $\mathrm{w}_b = (h + r)^2$:

$$\mathrm{w}_b = (h + r)^2 = h^2 + 2hr + r^2$$

Thus, the weighted average height h_L can be written as:

$$h_L = \frac{\sum_{i=1}^{n} h_i * ({h_i}^2 + 2h_i r + r^2)}{\sum_{i=1}^{n} ({h_i}^2 + 2h_i r + r^2)} = \frac{\sum_{i=1}^{n} ({h_i}^3 + 2{h_i}^2 r + r^2 h_i)}{\sum_{i=1}^{n} ({h_i}^2 + 2h_i r + r^2)}$$

Step2: Analyze the Difference $(h_w - h_L)$

We aim to analyze and determine the sign of the difference:

$$\Delta h = h_w - h_L$$

Substitute the formulas for h_w and h_L:

$$\Delta h=\frac{\Sigma_{i=1}^{n}{h_i}^3}{\Sigma_{i=1}^{n}{h_i}^2}-\frac{\Sigma_{i=1}^{n}({h_i}^3+2{h_i}^2r+r^2h_i)}{\Sigma_{i=1}^{n}({h_i}^2+2h_ir+r^2)}$$

Combine the two fractions into a single expression:

$$\Delta h=\frac{(\Sigma_{i=1}^{n}{h_i}^3)(\Sigma_{i=1}^{n}({h_i}^2+2h_ir+r^2))-(\Sigma_{i=1}^{n}({h_i}^3+2{h_i}^2r+r^2h_i))(\Sigma_{i=1}^{n}{h_i}^2)}{\Sigma_{i=1}^{n}{h_i}^2(\Sigma_{i=1}^{n}({h_i}^2+2h_ir+r^2))}$$

Step3**:** Expand and Simplify the Numerator

Expand the numerator:

$$\text{Numerator}=(\Sigma_{i=1}^{n}{h_i}^3)(\Sigma_{i=1}^{n}({h_i}^2+2h_ir+r^2))-(\Sigma_{i=1}^{n}({h_i}^3+2{h_i}^2r+r^2h_i))(\Sigma_{i=1}^{n}{h_i}^2)$$

Further expand and simplify, eliminating the common terms:

$$=2r\left(\Sigma_{i=1}^{n}{h_i}^3\Sigma_{i=1}^{n}h_i-\Sigma_{i=1}^{n}{h_i}^2\Sigma_{i=1}^{n}{h_i}^2\right)+r^2(\Sigma_{i=1}^{n}{h_i}^3-\Sigma_{i=1}^{n}h_i\Sigma_{i=1}^{n}{h_i}^2)$$

Step4: Determine the Sign

To determine the sign of Δh, consider the two parts:

First part:

$$2r\left(\Sigma_{i=1}^{n}{h_i}^3\Sigma_{i=1}^{n}h_i-\Sigma_{i=1}^{n}{h_i}^2\Sigma_{i=1}^{n}{h_i}^2\right)$$

Since r≥0, we need to analyze the sign of the term inside the parentheses. By applying the Cauchy-Schwarz inequality:

$$\Sigma_{i=1}^{n}{h_i}^2\Sigma_{i=1}^{n}{h_i}^4\geq(\Sigma_{i=1}^{n}{h_i}^3)^2$$

Thus:

$$\Sigma_{i=1}^{n}{h_i}^3\Sigma_{i=1}^{n}h_i\geq\Sigma_{i=1}^{n}{h_i}^2\Sigma_{i=1}^{n}{h_i}^2$$

So, the first part is non-negative.

Second part:

$$r^2(\Sigma_{i=1}^{n}{h_i}^3-\Sigma_{i=1}^{n}h_i\Sigma_{i=1}^{n}{h_i}^2)$$

Similarly, applying the Cauchy-Schwarz inequality:

$$\Sigma_{i=1}^{n}{h_i}^3\leq\sqrt{(\Sigma_{i=1}^{n}{h_i}^2)(\Sigma_{i=1}^{n}{h_i}^4)}$$

In general, for specific cases or for non-negative sequences, the original inequality:

$$\Sigma_{i=1}^{n}{h_i}^3\geq\Sigma_{i=1}^{n}h_i\Sigma_{i=1}^{n}{h_i}^2$$

can be demonstrated to hold using known inequalities or specific examples. The inequality can often hold true in practice or under specific conditions, but may not always be true in every case without additional constraints or conditions.

So, the second part is also non-negative.

Step5: Conclusion

Since the numerator is the sum of two terms, each of which is non-negative, and at least one of them is strictly positive (because r≥0), it follows that $\Delta h = h_w - h_L \geq 0$. Particularly, when the values h_i are not all equal, the difference is strictly greater than 0. The weighted average height $h_w \geq h_L$. Further, the weight $w_b = (h + r)^2$, which includes a positive linear term 2hr and a constant term r^2, resulting in higher weights for each h_i when calculating the weighted average. Consequently, as h_i increases, the difference between h_w and h_L also grows.

(2) Empirical Data Analysis

We validated our theoretical findings with empirical data. Our validation dataset, which includes measurements where DBH often exceeds tree height (Figure S1), supports the conclusion that $h_w \geq h_L$.

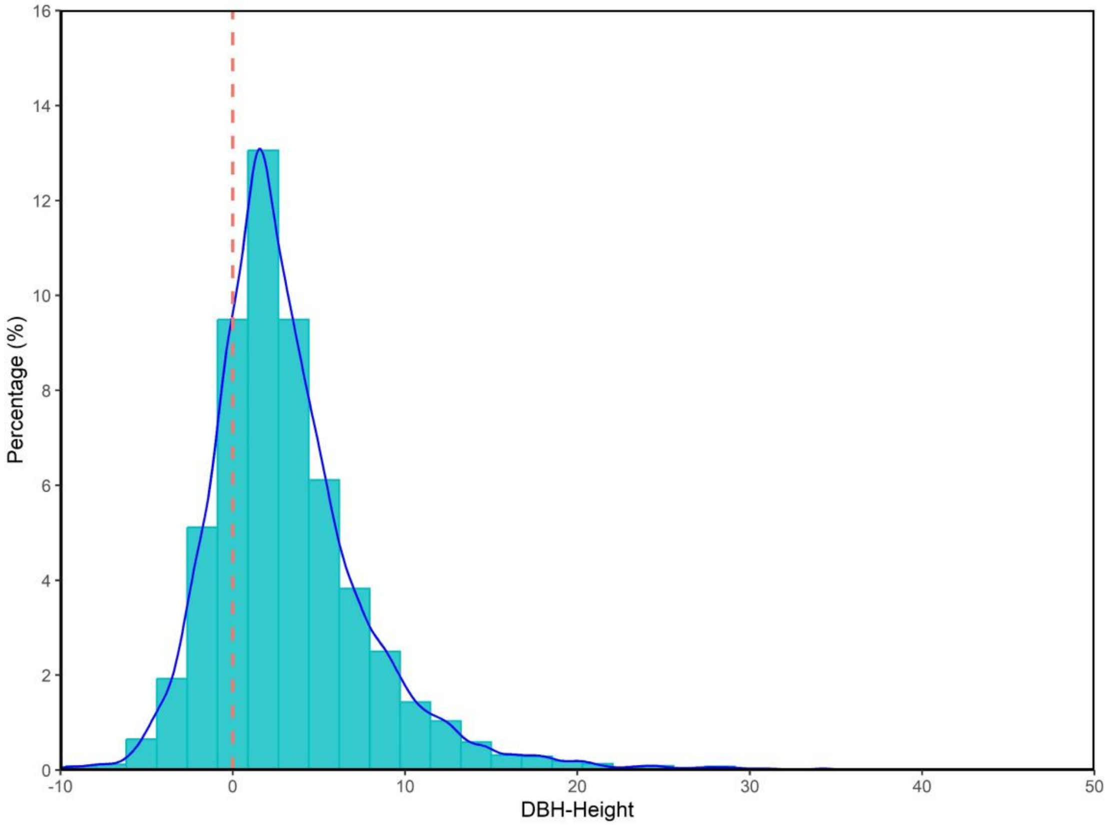

Figure S2: Frequency distribution of (DBH - Height) for tree measurement data in each plot

Please refer to supplementary note S2 and figure S2 for details.

Comment 7:

The decimal places of precision indexes in this paper should be consistent, such as Tabel

5.

Reply 7: We have adjusted to ensure the consistency of decimal places for the indexes. Please refer to Table 5 for details.