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Supplementary for \\ \section*{A global estimate of monthly vegetation and soil fractions from \\ \section*{A global estimate of monthly vegetation and soil fractions from spatio-temporally adaptive spectral mixture analysis during 2001 spatio-temporally adaptive spectral mixture analysis during 20012022}2022}

## Supplementary Methods

## Seasonal Mann-Kendall test

The seasonal Mann-Kendall test is commonly applied to identify trends for seasonal environmental data of interest that is available as time series for which the time intervals between adjacent observations arc less than one year (i.e., daily, weekly, and monthly sequences) (Hirsch et al. 1982). Letting the sequence $X$ consists of a complete seasonal record of $n$ year that includes $m$ seasons per year, the $X$ can be expressed by

$$
X=\left[\begin{array}{cccc}
x_{11} & x_{12} & \cdots & x_{1 m} \\
x_{21} & x_{22} & \cdots & x_{2 m} \\
\vdots & \vdots & & \vdots \\
x_{n 1} & x_{n 2} & \cdots & x_{n m}
\end{array}\right]
$$

The null hypothesis, $\mathrm{H}_{0}$, is that the $n$ observations come from each of $m$ seasons with independent realizations are identically distributed. While the alternative hypothesis $\left(\mathrm{H}_{\mathrm{A}}\right)$ of a two-sided test is that data presents a monotonic trend. The Seasonal Mann-Kendall test statistic for the $g$ th season is

$$
S_{g}=\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \operatorname{sgn}\left(x_{j g-} x_{i g}\right), g=1,2, \ldots, m
$$

where

$$
\operatorname{sgn}(\theta)=\left\{\begin{array}{c}
1 \text { if } \theta>0 \\
0 \text { if } \theta=0 \\
-1 \text { if } \theta<0
\end{array}\right.
$$

$S g$ is asymptotically normally distributed, thus the mean of $S g$ is $E\left[S_{g}\right]=0$, and the variance is

$$
\operatorname{Var}\left[S_{g}\right]=\left\{n(n-1)(2 n+5)-\sum_{j=1}^{p} t_{j}\left(t_{j}-1\right)\left(2 t_{j}+5\right)\right\} / 18
$$

where $n$ is the number of years of each season, $p$ is the number of tied groups for data $x_{i g}, i=1,2, \ldots n$, in season $g$, and $t_{j}$ is the number of data points in the $j$ th tied group. The seasonal Mann-Kendall test statistic is

$$
S=\sum_{g=1}^{m} S_{g}
$$

which is also asymptotically normally distributed where $E[S]=0$, thus the variance of $S$ is

$$
\operatorname{Var}[S]=\sum_{g=1}^{m} \operatorname{Var}\left[S_{g}\right]
$$

And the statistic $S$ is approximately normal distributed provided that the following Z-transformation is employed,

$$
Z=\left\{\begin{array}{l}
\frac{S-1}{\sqrt{\operatorname{Var}[S]}} \text { if } S>0 \\
0 \text { if } S=0 \\
\frac{S+1}{\sqrt{\operatorname{Var}[S]}} \text { if } S<0
\end{array}\right.
$$

For a given $\alpha$-significance level, the original null hypothesis $\left(\mathrm{H}_{0}\right)$ is unacceptable if $|\mathrm{Z}| \geq Z_{1-\alpha / 2}$. This implies a
significantly upward or downward trend in the series.

Theil-Sen estimator is a method of robust linear regression by selecting the median value of the slope of all lines passing through the paired points. It is also known as Sen's slope estimation (Sen and Kumar, 1968). Here, we detect slope of fractions according to seasonal Sen's method. For sequence $X$ consisting of a complete seasonal record of $n$ year that includes $m$ seasons per year, a set of linear slopes is calculated as,

$$
d_{g, j, k}=\frac{x_{g, j}-x_{g, k}}{j-k}
$$

For each $x_{i, j}, x_{i, k}$ pair $i=1,2, \ldots, m, 1 \leq k<j \leq n$, where $n$ is length of $g$ th season. and seasonal Sen's slope is then calculated as the median from all slopes.


Figure S1: The typical MODIS Grids for endmembers selection. a, 10 selected MODIS grids (i.e., h08v05, h12v12, h13v09, h16v01, h21v03, h22v02, h22v08, h24v06, h26v05, h27v06, h29v12), distributed in 6 continents, were colored with red. b, The Simpson's Diversity Index (D) greater than 0.6 for selected MODIS grids. $\mathbf{c}$, the correspondence between selected MODIS grids and KöppenGeiger climate classification, indicating each Köppen-Geiger climate classification was represented by selected MODIS grids.


Figure S2: Variance contributions (\%) of principal component transformation for each monthly MODIS reflectance image across $\mathbf{1 0}$ grids ( $\mathbf{n}=\mathbf{2 2 8 0}$ ). The violin plot and box plot revealed value distribution of each PC. The blue and red lines represented average of variance contributions and cumulative variance contributions of each PC, respectively. The cumulative contribution of the top three PCs has exceeded $99 \%$.


Fig. S3 Regional detailed subsets of changes for endmember fractions. From top to bottom, the first row represented composited images with $\Delta \mathrm{BS}, \Delta \mathrm{PV}$, and $\Delta \mathrm{DA}$. the second to sixth row displayed the change magnitude (\%) in each pixel for estimated endmembers, i.e., $\Delta \mathrm{PV}, \Delta \mathrm{NVP}, \Delta \mathrm{BS}, \Delta \mathrm{DA}$, and $\Delta \mathrm{IS}$. Pixels showing a statistically significant trend $(\mathrm{n}=228$, Seasonal Mann-Kendall test, $\mathrm{P}<0.05$ ) for either endmember are depicted on the change map. a-d, Eastern China, Western USA, Southern Asia, and Amazon.

Table S1 Selected typical MODIS grids for endmember selection and corresponding timing for each endmember.

|  | Girds Zones | IS | NPV | PV | BS | DA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| h08v05 | West Coast of North America | 2016-12 | 2005-02 | 2010-07 | 2005-02 | 2019-10 |
|  |  | 2009-12 | 2005-03 | 2012-07 | 2005-03 | 2007-10 |
|  |  | 2017-01 | 2010-03 | 2013-07 | 2010-03 | 2003-10 |
| h12v12 | West Coast of South America | 2003-06 | 2007-09 | 2007-01 | 2007-09 | 2003-04 |
|  |  | 2016-05 | 2011-09 | 2016-02 | 2011-09 | 2009-04 |
|  |  | 2005-08 | 2008-09 | 2014-02 | 2008-09 | 2013-04 |
| h13v09 | Amazon Basin | - | 2019-10 | 2016-01 | 2019-10 |  |
|  |  | - | 2013-10 | 2018-02 | 2013-10 |  |
|  |  | - | 2016-10 | 2004-03 | 2016-10 |  |
| h21h03 | Western Europe -Central Asia | 2018-02 | 2007-10 | 2014-09 | 2017-10 |  |
|  |  | 2011-02 | 2016-10 | 2013-09 | 2016-10 |  |
|  |  | 2005-03 | 2016-10 | 2004-09 | 2009-10 |  |
| h22h02 | Russian Far East | 2003-01 |  | 2019-06 | 2019-06 | 2006-03 |
|  |  | 2009-12 |  | 2018-06 | 2018-06 | 2011-03 |
|  |  | 2010-12 |  | 2019-07 | 2019-07 | 2012-03 |
| h22h08 | East Coast of North Africa | - |  | - | 2005-07 | 2013-04 |
|  |  | - |  | - | 2006-10 | 2018-05 |
|  |  | - |  | - | 2016-06 | 2019-10 |
| h24v06 | South Asia | - |  | 2018-07 | 2018-07 |  |
|  |  | - |  | 2004-07 | 2004-07 |  |
|  |  | - |  | 2008-07 | 2008-07 |  |
| h26v05 | Northwest of Qinghai Tibet Plateau | 2008-01 |  | 2013-07 | 2007-03 | 2007-09 |
|  |  | 2019-02 |  | 2006-07 | 2007-11 | 2017-09 |
|  |  | 2008-02 |  | 2010-07 | 2003-11 | 2008-10 |
| h27v06 | Southwest China | - |  | 2013-09 | 2007-11 |  |
|  |  | - |  | 2006-08 | 2019-10 |  |
|  |  | - |  | 2017-07 | 2004-10 |  |
| h29v12 | South Australia | - |  | 2019-08 | 2017-05 |  |
|  |  | - | 2003-01 | 2016-09 | 2011-05 |  |
|  |  | - | 2006-12 | 2016-08 | 2012-05 |  |



Table S2 692 combination models. These models include two-endmember model, three-endmember model and four-endmember model.

| Models | combinations |
| :---: | :---: |
| two-endmember model (88) | PV+BS (16) |
|  | PV+DA (8) |
|  | PV+IS (8) |
|  | BS+DA (8) |
|  | BS+IS (8) |
|  | DA+IS (4) |
|  | PV+NPV (12) |
|  | BS+NPV(12) |
|  | DA+NPV(6) |
|  | IS+NPV(6) |
| three-endmember model(252) | PV+BS+DA (32) |
|  | PV+BS+IS (32) |
|  | PV+DA+IS (16) |
|  | BS+DA+IS(16) |
|  | PV+BS+NPV (48) |
|  | PV+DA+NPV (24) |
|  | PV+IS+NPV (24) |
|  | BS+DA+NPV (24) |
|  | BS+IS+NPV(24) |
|  | DA+IS+NPV (12) |
| four-endmember model (352) | PV+BS+DA+IS (64) |
|  | PV+BS+DA+NPV (96) |
|  | PV+BS+IS+NPV (96) |
|  | PV+DA+IS+NPV(48) |
|  | BS+DA+IS+NPV (48) |

Table S3 Evaluation of estimated five vegetation and soil components against GLCVRD reference dataset.

|  | ME | MAE | RMSE | $\mathbf{R}^{\mathbf{2}}$ |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{P V}+\mathbf{N P V}$ | -0.100 | 0.118 | 0.149 | 0.592 |
| BS | 0.047 | 0.075 | 0.109 | 0.710 |
| DA | 0.047 | 0.050 | 0.065 | 0.156 |
| IS | 0.008 | 0.008 | 0.020 | 0.792 |

calculated for globe and five climate zones, i.e., tropical, arid, temperate, cold, and polar.

| Zones | Endmembers | Initial area | Loss | Gain | Net change area |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left(\mathrm{km}^{2}\right)$ | $\left(\mathrm{km}^{2}\right)$ | $\left(\mathrm{km}^{2}\right)$ | $\left(\mathrm{km}^{2}\right)$ |
| Tropical | PV | 49861610.43 | -637802.76 | 1573227.20 | 935424.44 |
|  | NPV | 15128296.27 | -564426.41 | 345275.53 | -219150.87 |
|  | BS | 46994793.90 | -1381062.00 | 866955.99 | -514106.02 |
|  | DA | 32708319.15 | -887176.16 | 660346.40 | -226829.76 |
|  | IS | 22247774.92 | -114679.83 | 139342.04 | 24662.21 |
|  | PV | 16018642.05 | -236558.32 | 231221.99 | -5336.33 |
|  | NPV | 2217754.40 | -109780.44 | 24544.40 | -85236.04 |
|  | BS | 3830525.28 | -197155.41 | 319038.83 | 121883.42 |
|  | DA | 2001545.61 | -61499.28 | 29922.32 | -31576.96 |
|  | IS | 133822.65 | -3499.33 | 3765.24 | 265.91 |
|  | PV | 9393375.78 | -78191.98 | 413284.59 | 335092.61 |
|  | NPV | 2417754.40 | -154026.78 | 52210.12 | -101816.66 |
|  | BS | 3530525.28 | -316382.59 | 102279.47 | -214103.11 |
|  | DA | 3001545.61 | -79412.55 | 62801.35 | -16611.20 |
|  | IS | 133822.65 | -9419.53 | 6857.89 | -2561.64 |
|  | PV | 7302896.73 | -166255.40 | 431469.49 | 265214.10 |
|  | NPV | 2753166.01 | -158937.90 | 149897.07 | -9040.83 |
|  | BS | 34208215.02 | -577137.23 | 402055.35 | -175081.89 |
|  | DA | 6055362.59 | -301342.53 | 226780.57 | -74561.96 |
|  | IS | 3234817.92 | -79628.12 | 73098.70 | -6529.42 |
|  | Cold | PV | 15661149.01 | -153482.53 | 488284.80 |
|  | NPV | 6556417.74 | -140497.42 | 114025.71 | -264802.27 |
|  | BS | 3772331.10 | -278401.74 | 38748.72 | -239653.02 |
|  | DA | 17529802.86 | -438961.80 | 334040.19 | -104921.61 |
|  | IS | 9580056.20 | -17785.65 | 54029.72 | 36244.07 |
|  | PV | 1485546.86 | -3314.53 | 8966.33 | 5651.80 |
|  | NPV | 1183203.72 | -1183.88 | 4598.24 | 3414.36 |
|  | BS | 1653197.22 | -11985.03 | 4833.61 | -7151.42 |
|  | DA | 4120062.48 | -5960.00 | 6801.97 | 841.96 |
|  | IS | 9165255.50 | -4347.20 | 1590.50 | -2756.70 |

