Review of ESSD-2020-246

This paper presents in my view a valuable contribution to the rescue of old geophysical data - in this case from early lidar measurements of stratospheric aerosols - for the sake of their use in the reconstruction of past volcanic events. The work is a contribution to the Data Rescue activity of the Stratospheric Sulfur and its Role in Climate within the SPARC project.

The crux of the work is the extraction of aerosol extinction coefficients at 532 nm between 12 km and 24 km from backscattering-ratio results at 694 nm retrieved in that range, under simplifying hypotheses, from lidar measurements carried out in different periods of 1964 at two different locations (Lexington, Massachusetts, and College, Alaska).

While the "translation" from the original results (the backscattering ratios at a given wavelength under the mentioned simplifying hypotheses) to the extracted ones (the aerosol extinction coefficients at another wavelength and correcting for the simplifications) is carefully explained, I found apparent inconsistences and ambiguities in the developed formulation, as well as in the notation, that the authors should explain or, if my concerns are proven right, correct.

I. The notation ambiguities are the following:

1. The symbol N_A is used with two different meanings. In Eq. (1) $N_A(z)$ is the molecular number density at altitude z. In Eq. (4) it is used for Avogadro's number.

2. Two different symbols, $SR_o(\lambda,z)$ and $SR(\lambda,z)$, are used for the backscattering ratio without a clear reason for establishing a difference. This is apparent in Eq. (6), where the $\beta_a^A(z)$ expression is said to be derived from Eq. (2), but the symbol $SR_o(\lambda,z)$ appears instead of the symbol $SR(\lambda,z)$ used in Eq. (2).

3. The notation T_{2w}^2 for the two-way atmospheric transmittance is in my view redundant: the two-way is implicit in the squared superscript. The subscript should be left to denote the origin of the transmittance, as is generically done in Eq. (8). Note, related to this, that in line 180 T_T^2 seems to be used with the same meaning as T_{2w}^2.

4. Letting aside possible corrections arising from the reasoning in point #2 sin section III of this review below, on possible inconsistences in the formulation, I think that there are possible ambiguities in the notation that should be clarified. For example, β_a^A is used to denote the aerosol attenuated backscatter coefficient, which is subsequently corrected by the molecular and ozone transmittances in Eq. (7). The result is called β_a, which seems to imply that this is the final aerosol backscatter coefficient, from which using the factor BEc(z, t) (by the way, what does the variable t mean?) the aerosol extinction coefficient α_a is derived. (Eq. (9)). But then one discovers that α_a has still to be corrected for the aerosol transmittance, the final aerosol extinction coefficient being called α_a^{Ta}. Although

this is a minor remark, I think it would be clearer to reserve the symbols β_a^{Ta} , α_a^{Ta} for the coefficients yet to be corrected for the aerosol transmittance and to use β_a, α_a for the final, fully corrected extinction coefficient.

II. With respect to the inconsistences in the formul developments:

1. While Eq. (1) is actually found in ref. **G-66** (Eq. (4.2) of this reference), it should be noted that this equation refers to the "expected signal from a molecular atmosphere" (page 50 of **G-66**), as it is made clear by the sentence (also in page 50 of **G-66**): "Thus, to derive the dust profiles, it is necessary to evaluate the intensity of the echoes for a dust-free atmosphere by using equation 3.8 for the case of Rayleigh scattering by air molecules." Therefore the statement on line 144 of the paper under review referring to Eq, (1): "The average photoelectron flux registered by the photomultiplier was represented by the expression:" is misleading: the average photoelectron flux was actually represented by $\frac{dn(z)}{dt} = K\frac{\beta}{z^2}$ (Eq. (3.8) of **G-66**, with K condensing all the multiplicative constants in that equation), Eq. (1) corresponding to the expected photoelectron flux from a molecular (reference) atmosphere.

2. The authors seem to imply that the scattering ratio SR$_o$(λ,z) obtained in ref. **G-66** corresponds to the expression in Eq. (2). However, following the data processing steps described in the paper I don't arrive at that expression. I explain in detail my understanding of the steps described in the manuscript to sustain my statement. For clarity, I keep the authors' notation notwithstanding my remark #3 on the notation in section I above:

a) To begin with, the photoelectron flux will actually be, taking into account the atmospheric transmittance,

$$\frac{dn(z)}{dt} = K\frac{\beta(\lambda,z)T_{2w}^2(z)}{z^2} = K\frac{\left[\beta_m(\lambda,z)+\beta_a(\lambda,z)\right]T_{2w}^2(z)}{z^2},$$

with $\beta(\lambda.z) = \beta_m(\lambda,z)+\beta_a(\lambda,z)$ the total backscatter coefficient, being $\beta_m(\lambda,z)$ and $\beta_a(\lambda,z)$ the molecular and aerosol backscatter coefficients respectively. Note that I'm using $\beta_a(\lambda,z)$ for the "true" (i.e. not affected by any attenuation) aerosol backscatter coefficient.

b) According to the authors: "Next, the ratios between the averaged signal at each level and the values at the same level of the right side of the equation (1) were calculated for each profile between 12 and 30 km". Calling that ratio P_n, I understand that

$$P_n(z)=\frac{\frac{dn(z)}{dt}}{\frac{N_A(z)}{z^2}}=K\frac{\left[\beta_m(\lambda,z)+\beta_a(\lambda,z)\right]T_{2w}^2(z)}{\beta_m(\lambda,z)}$$

c) Following the authors: "A final step consisted in normalizing the ratios calculated in each profile between 12 and 24 km. To that end, for each profile the average value between 25 and 30 km of the ratios calculated in the former step were determined". I will call the average value between 25 and 30 km $\overline{P}_n(z_1,z_2)$, with $z_1 = 25$ km and $z_2 = 30$ km. Then, assuming, as **G-66** seem to do, that $\beta_a(\lambda,z)$ is negligible in that range, we would have

$$\overline{P}_n(z_1,z_2)=\frac{1}{z_2-z_1}\int_{z_1}^{z_2}P_n(z)dz=\frac{K}{z_2-z_1}\int_{z_1}^{z_2}T_{2w}^2(z)dz=KT_{2w}^2(z_0),$$

with, according to the finite-increment theorem, z_0 a range between z_1 and z_2 whose value will depend on the form of $T_{2w}^2(z)$.

d) Always following the authors: "Then for each profile the ratios in the altitude range 12 and 24 km were divided by the average value of the ratios between 25 and 30 km from the same profile". Calling, as the authors seem to do, SR(λ,z) (or SR_0(λ,z), see my remark #2 on the notation ambiguities) the result of dividing the ratio P_n in the range 12 km – 24 km by $\overline{P}_n(z_1,z_2)$. Then

$$SR(\lambda,z)=\frac{P_n(z)}{\overline{P}_n(z_1,z_2)}=\frac{\left[\beta_m(\lambda,z)+\beta_a(\lambda,z)\right]T_{2w}^2(z)}{\beta_m(\lambda,z)T_{2w}^2(z_0)},$$

which do not coincide with the result given in Eq. (2).

The authors should, either point out possible mistakes I may have committed in the above development, or else correct theirs.

4. I couldn't understand the (iterative?) process described in lines 211 – 217 to obtain α_a^{Ta}. How is the $T_a^2(532,z)$ first guess of obtained? How is it refined? I suggest illustrating the procedure with a graph.

III. Other remarks

1. Line 77: "were been produce" should be just "were produced".

2. Line 80: "Giogio Fiocco" should be "Giorgio Fiocco".

3. Line 86: What do CMIP5 and CMIP6 refer to? Further explanations and possibly a reference are needed.

4. Line 137: "It is well known that solving the equation for the single wavelength elastic lidar is an ill-posed problem". What is the equation the sentence refers to? I suppose it is the lidar equation, so the sentence should read "It is well known that solving the lidar equation…" As implied in remark # 1 of section II of this review, this should be Eq. (1), but written in the form of Eq. (3.8) of **G-66.**

5. Line 105: the description "very high frequency nano-second laser" is misleading, as it seems to mix the laser pulse duration with the pulse repetition rate, the latter being very low, according to table 1, by today's standards.

6. I think it would be beneficial for the reader to point out that $\frac{dn(z)}{dt}$ is proportional to the collected backscattered power from that range z.

7. Line 146: n_z should probably be $n(z)$.

8. The references Jäger and Deshler, 2002 and 2003 are missing in the reference list. Please check that all cited references are included in the list.

9. Line 181: "Hosteler" should be "Hostetler"

10. In lines 186-187 it is said: "after neglecting the dispersion of the refractive index and the King factor of the air represented by k_{bw}". If the King factor is not used, it is unnecessary to assign it a symbol.

11. Unnecessary bracketing is used sometimes. For example, but not only, Eq. (2) could simply be written as

$$SR(\lambda,z)=\frac{\beta_m(\lambda,z)+\beta_a^A(\lambda,z)}{\beta_m(\lambda,z)},$$

without the curly brackets. Other instances of unnecessary brackets are found throughout the paper.

12. Lines 321-322: "After the conversion to 532 nm they were respectively 0.087 and 0.242…" What does "they" in this sentence refers to?

13. Line 326: "The high TAOD values for the twos series…" Which ones are the two series?

14. Lines 367-368: "the data we rescue are a reasonable approximation of what we today know as the backscattering ratio described in equation (2)". But the definition in Eq. (2), letting aside the concerns expressed in point #2 of section II of this review as to its derivation, uses the attenuated backscatter, while, in my understanding, the present-day backscattering ratio definition is $SR = \frac{\beta_m + \beta_a}{\beta_m}$, with β_m the molecular backscatter coefficient and β_a the "true", not affected by any attenuation, aerosol backscatter coefficient.