



1 **Heuristic Approach to Multidimensional Temporal Assignment**
2 **of Spatial Grid Points for Effective Vegetation Monitoring and Land**
3 **Use in East Africa**
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13 **Abstract:** In this research, vegetation trends are studied to give valuable information toward effective land use in
14 the East African region, based on the Normalized Difference Vegetation Index (NDVI). Previously, testing
15 procedures controlling the rate of false discoveries were used to detect areas with significant changes based on
16 square regions of land. This paper improves the assignment of grid points (pixels) to regions by formulating the
17 spatial problem as a multidimensional temporal assignment problem. Lagrangian relaxation is applied to the
18 problem allowing reformulation as a dynamic programming problem. A recursive heuristic approach with a
19 penalty/reward function for pixel reassignment is proposed. This combined methodology not only controls an
20 overall measure of combined directional false discoveries and nondirectional false discoveries, but make them as
21 powerful as possible by adequately capturing spatial dependency present in the data. A larger number of regions are
22 detected, while maintaining control of the mdFDR under certain assumptions.

23 Data Link: <https://figshare.com/s/ed0ba3a1b24c3cb31ebf>

24 DOI:

25 https://figshare.com/articles/NDVI_and_Statistical_Data_for_Generating_Homogeneous_Land_Use_Recommendations/5897581
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28 **Keywords:** Land Use, Mathematical Programming, Dynamic Programming, Multiple Testing, Spatial Data and
29 Analysis, False Discovery Rate

30

31 **1 Introduction**

32 Analysis of vegetation life cycles is fundamental in monitoring and planning agricultural endeavors and optimizing
33 land use. In particular, gaining knowledge of current vegetation trends and using them to make accurate predictions
34 is essential to minimize times of food scarcity and manage the consumption of natural resources in underdeveloped
35 countries. Needing to understand the Earth's ecology and land cover is increasingly important as the impacts of
36 climate change start to affect animal, plant, and human life. Vegetation trends are also closely related to
37 sustainability issues, such as management of conservation areas and wildlife habitats, precipitation and drought
38 monitoring, improving land usage for livestock, and finding optimum agriculture seeding and harvest dates for
39 crops.

40 For this reason, there are many agencies and organizations that focus on the study of land use and land cover trends,
41 linking them to climate change and the socioeconomic consequences of these changes. The United States Global
42 Change Research Program (Land Use and Land Cover Change Interagency Working Group), the United Nations
43 Framework Convention on Climate Change (Land Use, Land Use Change, and Forestry), and NASA's Land Cover
44 Land Use Change Program are just three examples of well-known interdisciplinary/ interagency programs that
45 conduct and sponsor research related to the question of global land change as noted in OCHA (2011).

46 Assessment of changes in a region's vegetation structure is challenging, especially in topographically diverse areas,
47 like East Africa. Forecasting future vegetation and agricultural planning become particularly difficult when



48 unknown trends are occurring. However, the regions with vegetation changes are often the areas of most interest in
49 land use management. Ideally, an automated screening process can identify areas with significant vegetation
50 changes and facilitate objective decision making about land-use management such as in Cressie & Wikle (2011).

51 As a first step in creating an automatic screening processes, data collection on vegetation and land cover is needed.
52 This is typically done through satellite remote sensing. The remote sensing imagery is used to convert the observed
53 elements (i.e., the image color, texture, tone, and pattern) into numeric quantities at each pixel in the image. The
54 image pixels correspond to a square grid of land, the size of which depends on the satellite's resolution. One such
55 numeric indicator is the normalized difference vegetation index (NDVI). In this article, the NDVI series came from
56 satellite remote sensing data collected between 1982 and 2006 over 8,000-meter grid points. It has been shown to be
57 highly correlated with vegetation parameters such as green-leaf biomass and green-leaf area, and hence is of
58 considerable value for vegetation monitoring as in Curran (1980) and Jackson, Et al. (1983).

59 The NDVI standard scale ranges from -1 to 1 , indicating how much live green vegetation is contained in the
60 targeted pixel. An NDVI value close to 1 indicates more abundant vegetation. For example, low values of NDVI
61 (say, 0.1 and below) correspond to scarce vegetation consisting mostly of rock, sand and dirt. A range of moderate
62 values (0.2 to 0.3) indicates small vegetation such as shrub or grassland; larger NDVI values can be found in
63 rainforests (0.6 to 0.8). Often, negative NDVI values are consolidated to be zero since negative values indicate non-
64 vegetation and are of little use for vegetation monitoring. Vegetation activity is a continuous space-time process and
65 NDVI data provide a space-time lattice system, in the sense that observations are available over equally spaced
66 regular grids. Often, the spatial resolution ranges from 1000 to 8000 meters, while the temporal one ranges from 7
67 days to 1 month.

68 Statistical and computational methods are needed to analyze remotely sensed data, like NDVI values, to determine
69 trends in land condition and to predict areas at risk from degradation. Methodologies that detect land cover changes
70 need to be sensitive as well as accurate, since it can be costly and risky to relocate human populations, agriculture or
71 livestock to new regions of detected change. In such spatio-temporal data, time series models are tempting for
72 representing such processes. Other existing change detection methodologies include the geographically weighted
73 regression of Foody (2003), the principal component analysis of Hayes & Sader (2001), and the smoothing
74 polynomial regression of Chen & Tamura (2004). However, these methods are unable to provide an upper bound on
75 false detections. Since there is large risk associated with falsely declaring an area to have significant vegetation
76 changes, land use managers seek new methods that have a meaningful control over such errors.

77 In this article, we build on the previous work of Vrieling, et al. (2008) and Clements, et al., (2014). Vrieling, et al.
78 (2008) first investigated this vegetation screening problem in the hypothesis testing framework of but did not
79 attempt to address the inherent multiplicity issue by controlling an overall false detection rate while making their
80 final conclusions. Clements, et. al. (2014) made improvements by incorporating the spatial dependencies, somewhat
81 arbitrarily, before applying multiple testing procedures. The arbitrary spatial dependency was accounted for by
82 dividing the region into square blocks, based on an overall measure of spatial correlation using a semivariogram



83 plot. After creating such sub-regions, two-sided monotonic trend tests from Brillinger (1989) were used to identify
84 significant increasing or decreasing monotonic vegetation changes based on these arbitrarily chosen square regions
85 of land. They demonstrated that this screening procedure controlled the mixed directional false discovery rate
86 (mdFDR), which is defined as the expected proportion of Types I errors (False Positives) and Type III errors
87 (Directional errors) among all rejected null hypotheses, introduced by Benjamini & Yekutieli (2005).

88 In this article, we utilize the same historic NDVI time series for East Africa from 1982 to 2006. Since real-time
89 monitoring for change is not part of the scope, we focused improving the methodologies previously used to identify
90 significant changes in land cover in the region. We do this by first framing the research question as an NP-hard
91 temporal multi-objective assignment problem. Using heuristics to solve this problem, we first find improved sub-
92 regions than the previous arbitrarily chosen square grids. Using this approach allows us to adequately capture the
93 specific data structure and answer questions in the present context. Secondly, we reapply the multiple testing
94 procedures in Clements, et al., (2014) and demonstrate that the testing procedure become more powerful while still
95 maintaining control an error rate, the mdFDR. In summary, our methods aim to incorporate spatial local
96 dependencies using a multi-dimensional assignment problem formulation to improve sub-region formation, which in
97 turn improves the multiple testing results.

98 We organize the paper as follows. In the next section, we give a review of the literature followed by a detailed
99 description of the historical data set. We then describe the temporal assignment problem formulation to create more
100 homogeneous sub-regions and explain the heuristic procedure using dynamic programming. Next, we apply the
101 multiple testing procedures to the improved sub-regions. Finally, we reveal the results of the model implementation,
102 followed by a discussion, conclusions, and final remarks.

103 **2 Literature**

104 **2.1 Multiple Testing Overview**

105 To control over false vegetation trend detections, multiple testing procedures can be employed. An overview of
106 multiple testing notation and procedures are described next. When testing a single null hypothesis against a two-
107 sided alternative, two types of error can occur when a directional decision is made following rejection of the null
108 hypothesis. These are Type I error and Type III (or directional) errors. The Type I error occurs when the null
109 hypothesis is falsely rejected, while the Type III error occurs when the null hypothesis is correctly rejected but a
110 wrong directional decision is made about the alternative.

111 Consider testing n hypotheses simultaneously, such as testing for trend changes in n pixels over the East African
112 region. Table 1 gives the various outcomes of these tests, where $H_{i0}: \theta_i = \theta_{i0}$ is the null hypothesis and $H_{i1}: \theta_i \neq$
113 θ_{i0} is the two-sided alternative, for $i = 1, 2, \dots, n$. Of these quantities in Table 1, only n, A , and R (where $R = R_1 +$
114 R_2) are known after applying a particular testing procedure. The number of Type I errors, Type II errors, and Type
115 III errors are $V = V_1 + V_2$, $T = T_1 + T_2$, and $U = S_2 + S_3$ respectively. All three quantities are unknown but



116 desirably small. Most multiple testing procedures focus on controlling V in some capacity. In this paper, we utilize a
 117 procedure that controls V and U .

		Decision			Total
		Fail to Reject Null	Reject Null $H_0^{(+)}$	Reject Null $H_0^{(-)}$	
Truth	$\theta = \theta_0$	W (Correct Decisions)	V_1 (Type I errors)	V_2 (Type I errors)	n_0
	$\theta > \theta_0$	T_1 (Type II errors)	S_1 (Correct Decisions)	S_2 (Correct Decisions)	n_+
	$\theta < \theta_0$	T_2 (Type II errors)	S_3 (Correct Decisions)	S_4 (Correct Decisions)	n_-
Total		A	R_1	R_2	n

118

119

Table 1: Multiple Testing outcomes from testing n hypotheses

120 One of the most commonly used measures of overall Type I error is called the Familywise Error Rate (FWER). The
 121 FWER is the probability of making one or more Type I errors. In other words, out of n simultaneously tested
 122 hypotheses, where V is the number of Type I errors made out of n decisions (recall: V is an unknown quantity), then
 123 $\text{FWER} = \text{Prob}\{V > 0\}$. In the case of multiple hypothesis testing, the FWER should be controlled at a desired
 124 overall level, called α . The Bonferroni procedure is the most popular method to control the FWER, but there are
 125 other techniques, such as those in Holland & Copenhaver (1987), Hochberg & Tamhane (1987), Šidák (1967), Holm
 126 (1979), Hochberg (1988), Sarkar (1998), and Sarkar & Chang (1997).

127 The False Discovery Rate (FDR), proposed by Benjamini and Hochberg (1995), is the second most common
 128 measure of Type I errors. The FDR is the expected proportion of Type I errors among all the rejected null
 129 hypotheses. If there are no rejected hypotheses, the FDR is defined to be zero. In terms of Table 1, $\text{FDR} =$
 130 $E\left[\frac{V}{\max(R, 1)}\right]$. Comparatively, the FDR is less conservative than the FWER, meaning FWER control ensures
 131 FDR control. However, a multiple testing procedure with FDR control will not necessarily maintain control of the
 132 FWER. The FDR is a widely accepted and utilized notion of Type I errors in large-scale multiple testing
 133 investigations. Recent literature has proposed methods to control the FDR, including Benjamini and Hochberg
 134 (1995), Benjamini and Yekutieli (2001), Sarkar (2002), Blanchard and Roquain (2009), Storey, Taylor, and
 135 Siegmund (2004), and Benjamini, Krieger, and Yekutieli (2006).

136 Often, it becomes essential for researchers to determine the direction of significance, rather than significance alone,
 137 when testing multiple null hypotheses against two-sided alternatives. In other words, for each test, researchers have
 138 to decide whether or not the null hypothesis should be rejected and, if rejected, determine the direction of the
 139 alternative. Typically, this direction is determined based on the test statistic falling in the right- or left-side of the
 140 rejection region. Such decisions can potentially lead to one of two types of error for each test resulting in rejection of
 141 the null hypothesis - the Type I error if the null hypothesis is true or the directional error, also known as the Type III



142 error, if the null hypothesis is not true but the direction of the alternative is falsely declared (i.e. a rejection of a false
143 null using a two-sided alternative, but where the sign of the true parameter, say β_i , is opposite of its estimate $\hat{\beta}_i$).

144 Two variants to deal with Type I and Type III errors have been introduced in the literature. First is the pure
145 directional FDR (dFDR), which is the expected proportion of directional errors among rejected hypotheses. Second
146 is the mixed directional FDR (mdFDR), which is the expected proportion of Type I and Type III errors among
147 rejected hypotheses. To deal with both errors in an FDR framework, the notion of mixed directional FDR (mdFDR)
148 was been introduced by Benjamini et al. (1993). Since then, other methods to control directional errors have been
149 introduced, including Benjamini and Yekutieli (2005), Benjamini and Hochberg (2000), Shaffer (2002), Williams et
150 al. (1999), Guo et al. (2009), and Sarkar and Zhou (2008).

151 Controlling both false discoveries (V , from Table 1) and directional false discoveries (U , from Table 1) is important
152 in this application. For instance, when declaring a particular $8,000 \text{ m} \times 8,000 \text{ m}$ grid of land as ‘significantly’
153 changing in terms of vegetation, a Type I error is made if the area is not truly changing, and a Type III error is made
154 if the area is truly changing but in the opposite direction of what is determined from the data. When such decisions
155 are made simultaneously based on testing multiple hypotheses, one should adjust for multiplicity and control an
156 overall measure of Types I and III errors. Without such multiplicity adjustment, more Types I and III errors can
157 occur than the desired α level. It is particularly important to avoid these errors as much as possible in the present
158 application. Land use managers, government and local farmers are looking to relocate East African populations of
159 people, livestock and crops to areas of promising vegetation changes and avoid regions with decreasing changes.
160 Since these migrations can be risky and costly, a careful consideration of the multiplicity issue seems essential when
161 making declarations of significant vegetation changes.

162 In this article, p-values generated using the monotonic trend test in Brillinger (1989) are computed for each site
163 ($8,000 \text{ m} \times 8,000 \text{ m}$ grid of land) and provide evidence of vegetation change occurring over the years—the smaller
164 the p-value, the higher is the evidence of a significant vegetation change. For each site, a decision must be made
165 regarding the significance of vegetation change that might have occurred over the years at that site, and, if
166 vegetation change is found significant, determine the direction in which this change has taken place. This must be
167 done simultaneously for all sites ($\approx 50,000$) in the East African region in a multiple testing framework designed to
168 ensure a control over a meaningful combined measure of statistical Types I and III errors.

169 In this paper, we will first be framing the research question as a heuristic multi-objective temporal assignment
170 problem, in which better sub-regions were created than the arbitrarily chosen square grids in Clements et.al. (2014).
171 By using temporal assignments to create subregions, we will demonstrate that the testing procedure becomes more
172 powerful. Also in this article, we provide theoretical proof that the mdFDR is still controlled under sub-region
173 independence.

174 **2.1 Temporal Assignment Problem Overview**



175 There is a wealth of research on assignment problems and specialized assignment problems that display
176 complicating constraints. Though the generalized assignment problem is solvable, once the number of dimensions
177 reaches 3, as in the formulation presented in this paper, this is no longer the case.

178 The multidimensional assignment problem was introduced by Pierskalla (1968) and a bibliography of multidimensional assignment problems was prepared by Gilbert & Hofstra (1988). Miori (2011, 2008, 2014) used
179 assignment problems to model truckload routing problems and the Pollyanna gift exchange problem. Scheduling
180 assignment problems to model truckload routing problems and the Pollyanna gift exchange problem. Scheduling
181 medical residents with the temporal component was addressed by Franz & Miller (1993). Bandelt, Et al. (1994,
182 2004) addressed multi-dimensional assignment problems with decomposable costs. The three-dimensional
183 assignment problem was applied to teaching schedules by Frieze & Yadegar (1981) and Balas & Saltman (1991).
184 Multidimensional approximation was applied to capacity expansion problems by Troung & Roundy (2011).
185 Lagrangian Relaxation was applied to a multi-dimensional assignment problem arising from multi-target tracking by
186 Poore & Rijavec (1993). Multi-tracking data was also addressed by Robertson (2001).

187 Approximations to the multi-dimensional assignment problem were generated by Kuroki & Matsui (2007), Gutin,
188 Et al. (2008), Krokhmal, Et al. (2007), and Karapetyan & Gutin (2011). The multi-objective assignment problem
189 seeking solutions to the assignment problem in the face of additional objectives using efficient sets was posed by
190 White (1984). A weighting function approach has also been applied to multi-objective (multicriteria) problems with
191 conflicting objectives by Phillips (1987).

192 Agricultural planning problems have been addressed by Samuelson (1952), Takayama (1964), Norton & Scandizzo
193 (1981), Kutcher & Norton (1982), Önal & McCarl, and Weintraub & Romero (2017). Multicriteria approaches to
194 agriculture decisions have also been applied by Gasson (1973), Harper & Eastman (1980), Wheeler & Russel
195 (1977), Hayashi (2000), and Romero & Rehman (2003).

196

197 **2.2 Land Use Optimization Overview**

198 The most basic methods in land use optimization involve limited enumeration of alternatives and developing metrics
199 to directly assess these alternatives. Landscape metrics addressing various land use goals were used by Kuchma, Et
200 al. (2013) to evaluate enumerated options for land use. A similar approach was proposed by Wang & Guldman
201 (2015) to mitigate seismic damages in Taichung, Taiwan.

202 Heuristic methods and in sustainable land use were applied by Steward, Et al. (2004), Cao, Et al. (2011), Liu, Et al.
203 (2016) and Sahebgharani (2016). Genetic algorithms were presented Cao, Et al. (2012) and the Analytical
204 Hierararchy Process was utilized by Memarian, Et al. (2014). Multi-objective linear programming with sensitivity
205 analysis was found effective by Sadeghi, Et al. (2009) while Soil and Water Assessment Toll (SWAT) was
206 employed by Sunandar, Et al. (2014).



207 3 Data Description

208 East Africa spans a wide variety of climate types and precipitation regimes which are reflected in its vegetation
209 cover. To capture this, satellite imagery was collected over a sub-Saharan region of East Africa that includes five
210 countries in their entirety (Kenya, Uganda, Tanzania, Burundi and Rwanda) and portions of seven countries
211 (Somalia, Ethiopia, South Sudan, Democratic Republic of Congo, Malawi, Mozambique and Zimbabwe). This
212 roughly ‘rectangular’ region extends from 27.8°E to 42.0°E longitude and 15.0°S to 6.2°N latitude. Also included in
213 the region are several East African Great Lakes such as Lake Victoria, Lake Malawi and Lake Tanganyika.
214 Vegetative analysis in this region is of interest for a variety of reasons, including the importance of the region for
215 global biodiversity and the vulnerability of the region to climate change, deforestation of the Congo, urban
216 development, civil conflict, and agricultural practices.



217
218
219

Figure 1 The study area, as indicated by the box.

220 The remotely sensed images were recorded twice a month from 1982–2006 and then converted to NDVI values.
221 Hence, the spatio-temporal data set consists of approximately 50,000 sites (pixels), each with 600 time series
222 observations (24 observations per year over 25 years). The satellite’s resolution corresponds to each pixel spanning
223 an 8,000m × 8,000m grid of land, which we will refer to as a ‘location.’ This Global Inventory Modeling and
224 Mapping Studies (GIMMS) data set is derived from imagery obtained from the Advanced Very High Resolution



225 Radiometer (AVHRR) instrument onboard the National Oceanic and Atmospheric Administration (NOAA) satellite
226 series 7, 9, 11, 14, 16 and 17. The NDVI values have been corrected by Tucker, Et al. (2005) for calibration, view
227 geometry, volcanic aerosols, cloud coverage and other effects not related to vegetation change.

228 All the negative NDVI values were consolidated to zero, as commonly done in vegetation monitoring, and re-scaled
229 the remaining values by 1,000. Negative NDVI values indicate non-vegetation areas, and so they are of no use in our
230 statistical analysis. Prior to the analysis, we examined the data for quality assurance and eliminated a small number
231 of pixels that were found to have several consecutive years with identical data values, which may be due to data
232 entry errors or machine malfunction.

233 When this data was first examined in Vrieling, de Beurs and Brown (2008), the percentage of pixels with the trend
234 test p-value less than $\alpha = 0.10$ was reported separately for positive and negative slopes. The reported results indicate
235 that much of the region has ‘significant’ vegetation change. For example, the cumulative NDVI indicator detected
236 44.2% of sites with p-values less than 0.10. However, this result fails to address the important statistical issue of
237 multiplicity when making these claims about significant vegetation changes and their directions simultaneously for
238 all the regions based on hypothesis testing. Later, Clements, et. al. (2014) addressed the multiplicity issue by
239 proposing a 3-stage multiple testing procedure to control the mixed-directional False Discovery Rate (mdFDR), but
240 did so on subregions of East Africa that were not optimally formed.

241 The associated csv file for this analysis is the information generated from Clements, et al, (2014) which was the
242 initial starting point for this analysis. It contains the following fields:

- 243 • site: Consecutive ID number, acting as a unique identified
- 244 • xcoord: pixel longitude
- 245 • ycoord: pixel latitude
- 246 • ndvi.avg: Overall pixel average NDVI from 1982 to 2006 (observations taken twice monthly)
- 247 • pval: Resulting p value from the Brillinger Trend Test (Brillinger, 1989)
- 248 • slp: Resulting slope from the Brillinger Tren Test (Brillinger, 1989)
- 249 • block: Block number – initial assignment was arbitrary

250 Using the algorithm below, followed by the multiple testing procedure, users may generate the revised and improved
251 block assignments.

252

253 **4 Assignment Problem Formulation**

254 We propose an assignment formulation to this problem, using these analysis results, with the goal of an improved
255 solution. The object of the geographic assignment problem is to map each pixel within the satellite images to an
256 appropriate block based upon a target value for each block. The block target values represent equal size ranges
257 within the overall range of the objective function values. The objective function for the pixel assignment is the sum



258 of absolute difference between the pixel NDVI and the block target NDVI. The number of blocks is set objectively
 259 and may be reset for each assignment problem solution generated.

260 Note that pixels may be formed entirely of water; these pixels have been assigned arbitrarily high NDVI values to
 261 effectively eliminate them from consideration in the block assignments. A ‘water block’ with an arbitrarily high
 262 target value ensures that all of these pixels may be assigned to blocks.

263 The objective of the pixel assignment problem is to minimize the NDVI difference function. Let m = the number of
 264 pixels, let n = the number of blocks, and let T = the number of time periods. The decision variable x_{ij}^k is a binary
 265 variable that represents the assignment, or lack of assignment, of pixel i to block j at time k . The constraints
 266 formulated ensure that each pixel is assigned to a block, during each period of time. The formulation in Eq. (1) – (3)
 267 follows the notation.

$x_{i,j}^k$	Decision variable $\in (0,1)$ $i = 1, \dots, m; j = 1, \dots, n; k = 1, \dots, T$
N_i^k	Pixel i NDVI score for time k : $i = 1, \dots, m; k = 1, \dots, T$
N_j^k	Block j NDVI target for time k : $j = 1, \dots, n; k = 1, \dots, T$

268 **Table 2 Assignment Problem Notation.**

269

270
$$\text{Minimize } \sum_k \sum_j \sum_i |N_j^k - N_i^k| \cdot x_{i,j}^k \quad (1)$$

271 Subject to:

272
$$\sum_i \sum_j x_{i,j}^k = 1 \text{ for } k = 1, \dots, T \quad (2)$$

273
$$x_{i,j}^k \in (0,1) \quad (3)$$

274 The binary decision variables utilize three indices, rendering the problem NP hard. We therefore propose and
 275 employ a heuristic approach that relies heavily on dynamic programming.

276 **5 Assignment Problem Solutions**

277 **5.1 Lagrangian Relation**

278 Restatement of the pixel assignment problem as a Markov Process will facilitate alternative solution methodologies.
 279 We present a Lagrangian relaxation of the formulation and introduce a Lagrangian multiplier (φ_k) for the single
 280 constraint to be relaxed in each time period $k = 1, \dots, T$. We include a simplifying assumption that the penalty is
 281 constant over all time periods and is denoted as φ . The revised formulation is presented in Eq. (4) - (5).



282

$$\text{Minimize } \sum_k \sum_j \sum_i |N_j^k - N_i^k| \cdot x_{i,j}^k + \sum_k \varphi \left(\sum_j \sum_i x_{i,j}^k - 1 \right) \quad (4)$$

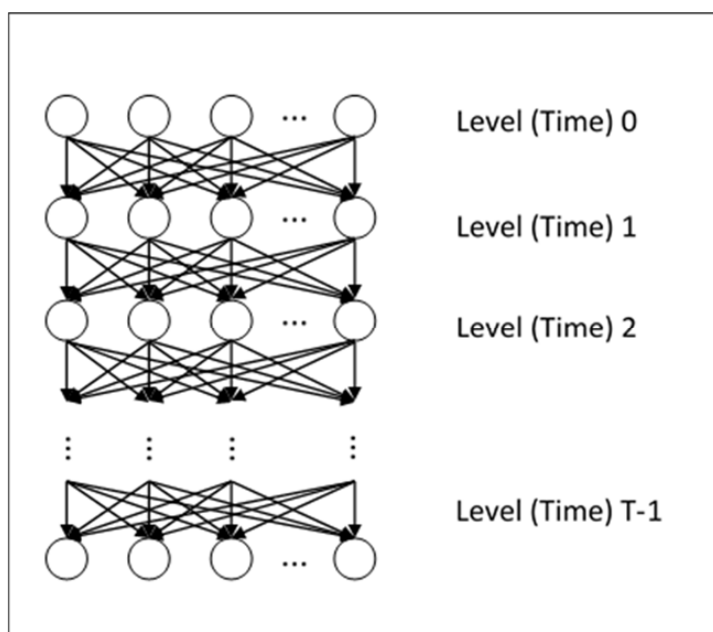
283 Subject to:

$$284 \quad x_{i,j}^k \in (0,1) \quad (5)$$

285 A dynamic programming formulation may now be presented using the relaxed formulation.

286 **5.2 Dynamic Programming Formulation**

287 The pixel assignment decisions may be made in stages, and while the outcome of each decision is not fully
 288 predictable, it can be observed before the next decision is made. We begin the dynamic programming formulation
 289 by organizing the problem into a tree structure (Fig. 2) reflecting pixels and levels (time increments). Each level of
 290 the tree corresponds to a time increment, beginning with time 0 which represents the first satellite images retrieved
 291 within the data set and ending at the final images at time T-1 and the pixels in each level number from 1 to m. The
 292 tree provides a discrete-time dynamic system.



293

294

295

Figure 2 General Tree Structure.



296 An additive value function reflects both present cost of each pixel assignment to a block, and potential future cost of
 297 all pixel assignments to blocks (expected cost-to-go). Block NDVI targets must be established in order to match
 298 pixels to blocks. Initialization of these targets is accomplished by evenly distributing the range of NDVI values
 299 across n candidate blocks. Recall that the NDVI values ranges between 0 and 1000, resulting in block targets
 300 starting at zero with an increment $1000/n$ up to 1000.

301 To calculate expected cost-to-go, we must also identify and calculate transition probabilities. In doing so, we
 302 consider only the current level (time period). The Markov Property (6) allows us to omit consideration of the
 303 probabilities of the path leading to the current level. The tree may now be viewed as a finite Nonhomogeneous
 304 Markov Process with transition probability matrix $P^{(k)}$ representing transitions at any level.

$$305 \quad P(X_{k+1} = x_{k+1} | X_0 = x_0, \dots, X_k = x_k) = P(X_{k+1} = x_{k+1} | X_k = x_k) \quad (6)$$

306 The objective of the dynamic programming formulation is the minimization of the sum of cost at the current stage,
 307 and the cost-to-go (the best case to be expected from future stages). The notation required for the formulation
 308 follows.

$A(k+1, k)$	Available pixels at level (time) $k+1$, depends on pixel chosen at level k
m_{k+1}	Cardinality of $A(k+1, k)$ (the number of pixels available at level $k+1$, depends on pixel selected at level k)
$s(k)$	The pixel chosen at level k
$P^{(k)}$	Transition probability matrix at level k
$P_{i,j}^{(k)}$	Transition probability of moving from pixel i to pixel j at level k
$C_{s(k),j}$	Cost of adding node j after the pixel chosen at level k
$U(i, k)$	The number of unassigned pixels if we choose pixel i at level k
$f(i, k)$	Expected cost-to-go if we choose pixel i at level k
$f(1,0)$	Initialize to 0
φ	Pixel assignment penalty

309

310 Pixel assignments to blocks may begin at any pixel in level 0 of the tree and end at any pixel in level $T-1$. All pixels
 311 must be assigned to a single block but individual blocks need not have pixels assigned to them. Let z be the
 312 candidate block.

$$313 \quad f(s(k), k) = \min_{z \in A(k+1, k), \varphi} \left\{ C_{s(k),z}^k + P_{i,j}^{(k)} \varphi U(s(k), k) + f(z, k+1) \right\} \quad (7)$$

$$314 \quad C_{s(k),z}^k = |N_z^k - N_{s(k)}^k| \quad (8)$$

315 Though this approach resolves issues with the original assignment formulation, it necessitates the calculation of
 316 transition probability matrices ($P^{(k)}$) at each level. Transition probabilities are dependent on the number of blocks
 317 chosen, and the ability to statistically characterize the changes in vegetation in the pixels over time. With as few as



318 100 blocks, the probabilities would have a very small order of magnitude and an expectation of high levels of
 319 inaccuracy, resulting in a lack of ability to detect meaningful differences. We present a heuristic, rooted in dynamic
 320 programming principles to render an efficient and useful solution to the pixel assignment problem.

321 5.3 Recursive Heuristic Procedure

322 Due to the original assignment problem being NP hard, and the dynamic programming approach resulting in
 323 extreme computational and structural complexity, we introduce a heuristic method that leverages knowledge gained
 324 in the assignment and dynamic programming approaches. This heuristic also leverages the previous research
 325 completed in controlling the mdFDR.

326 The heuristic procedure was initialized with the 150 blocks used in Clements et. al (2014) and 56,355 total pixels,
 327 and utilized the previously calculated slopes and resulting p-values from monotonic trend tests. Rather than
 328 assigning the pixels to blocks over the duration of the 25-year span of the data collection as the assignment
 329 formulation would, this approach focused on assignment at the final observations in the 25th year but the use of
 330 slope and p-value allowed the approach to reflect the trends that occurred over time. This same approach could be
 331 used at any time during the study, reflecting all previous data.

332 The heuristic performance metric, like the objective function in the pixel assignment problem, required the
 333 calculation of block values corresponding to the pixel values. The metric leverages the initial random blocks by
 334 including the block average NDVI, the block average slope, the block average p-value, and the slope change
 335 indicator variable. Notation is introduced in Table 3, followed by the formulation of the performance metric.

y_{fg}	Block assignment $\in (0,1) f = 1, \dots, m; g = 1, \dots, n$
I_{fg}	Slope change Indicator variable
N_f	Pixel i NDVI score at final observation: $i = 1, \dots, m$
NR_g	NDVI range for block g
S_f	Pixel i slope over time: $i = 1, \dots, m$
SR_g	Slope Range for block g
P_f	Pixel i p-value over time: $i = 1, \dots, m$
PR_g	p-value range for block g
w_d	Weight for scoring factor $d: d = 1, \dots, 4$

336 **Table 3 Heuristic Metric Notation.**

337 Let f = pixel number and let g = block number and let

$$338 y_{fg} = \begin{cases} 1 & \text{if pixel } f \text{ is assigned to block } g \\ 0 & \text{if pixel } f \text{ is not assigned to block } g. \end{cases}$$

339



340 Development of the performance metric required definition of the block average values for NDVI, slope and p-value
 341 shown in Eq (9) – (11). In addition, the indicator parameter, signaling slopes of opposite sign is shown in Eq. (12).

$$342 \quad \bar{N}_{.g} = \sum_{f \ni \text{pixels} \in \text{block } g} \frac{N_{fg}}{n} \quad \forall g = 1, \dots, n \quad (9)$$

$$343 \quad \bar{S}_{.g} = \sum_{f \ni \text{pixels} \in \text{block } g} \frac{S_{fg}}{n} \quad \forall g = 1, \dots, n \quad (10)$$

$$344 \quad \bar{P}_{.g} = \sum_{f \ni \text{pixels} \in \text{block } g} \frac{P_{fg}}{n} \quad \forall g = 1, \dots, n \quad (11)$$

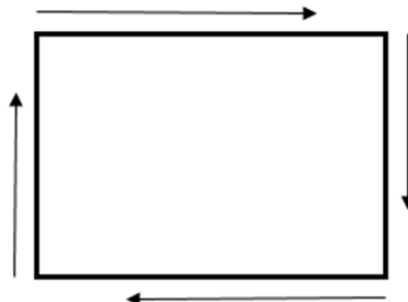
$$345 \quad I_{fg} = \begin{cases} 1 & \text{if } \text{sign}(S_f \cdot \bar{S}_{.g}) \text{ is negative } \forall f \ni \text{pixels} \in \text{block } g \\ 0 & \text{if } \text{sign}(S_f \cdot \bar{S}_{.g}) \text{ is positive } \forall f \ni \text{pixels} \in \text{block } g \end{cases} \quad (12)$$

346 The minimum value of the performance metric in Eq. (13) determines the highest quality heuristic solution. Pixels
 347 whose current assignment leaves them on the border between blocks are evaluated. The metric is calculated for their
 348 incumbent (current) assignment and their prospective assignment(s). The pixel is then assigned to the block yielding
 349 the lowest value of the metric. As pixels are reassigned, newly exposed border pixels are evaluated in the same
 350 fashion. This procedure continues until all border pixels belong to the block with the best fit.

$$351 \quad w_1 \frac{|N_f - \bar{N}_{.g}|}{NR_g} + w_2 \frac{|S_f - \bar{S}_{.g}|}{SR_g} + w_3 \frac{|P_f - \bar{P}_{.g}|}{PR_g} + w_4 I_{fg}$$

$$352 \quad \forall \text{ pixel } f = 1, \dots, m; \text{ bordering block } g = 1, \dots, n \quad (13)$$

353 The dynamic programming concept of forward and backward passes has been adapted for the heuristic to
 354 compensate for directional bias in the results. In this way, all border pixel assignments may be evaluated in all
 355 directions. Four starting points and starting directions are identified in Fig. 3. Fig. 4 shows the four passes to be
 356 completed for the first starting direction (upper left-hand corner). The first two passes are the forward direction
 357 evaluation and the second two passes are the backward direction evaluation. These same four passes are adapted for
 358 each starting point/direction, with the first pass always corresponding to the starting position.

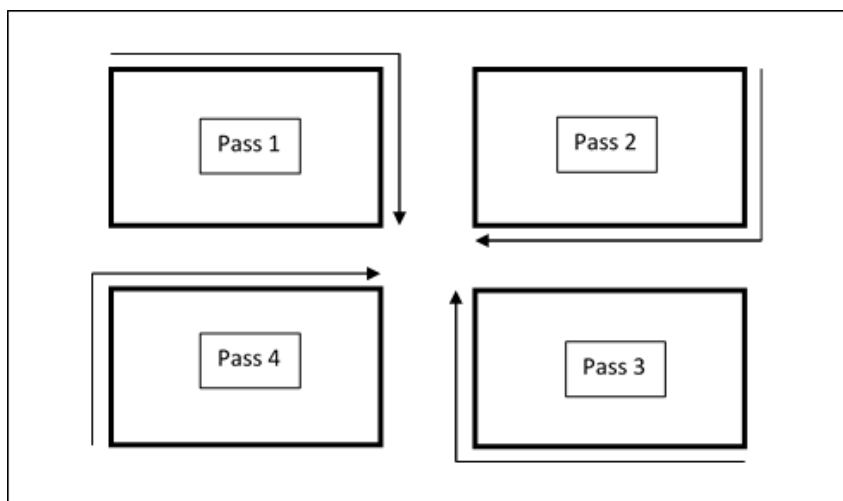


359



360
361

Figure 3 Starting Directions for Evaluation of Pixel Assignments.



362

Figure 4 Forward-backward evaluation: Forward passes 1 and 2; Backward passes 3 and 4.

363

364

365 Implementation and validation of the heuristic was accomplished through the development of a program written in
366 the C programming language.

367 6 Reassignment Model and Implementation

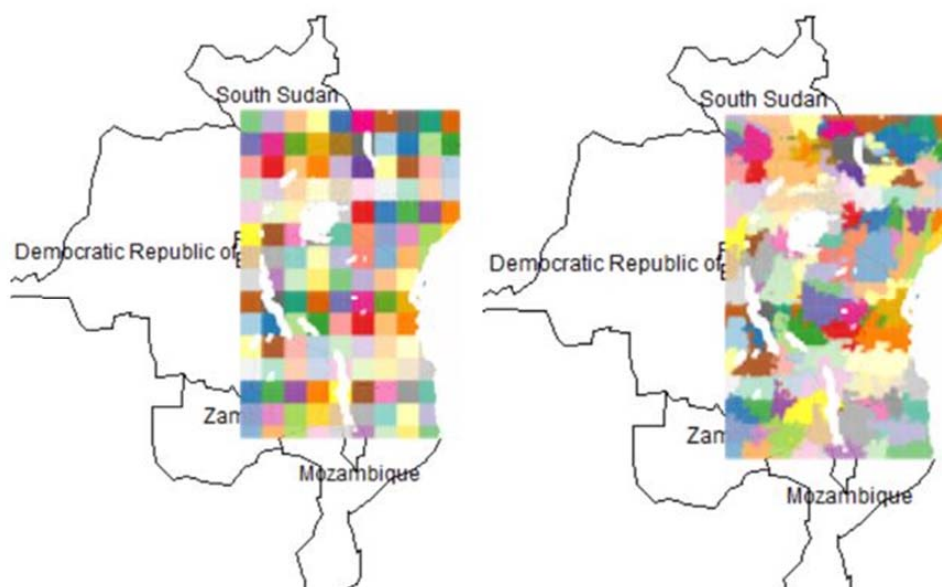
368 An approach inspired by dynamic programming was utilized to find the best solution to the heuristic problem based
369 on weight factors that varied between 0 to 1, under the condition that $\sum_{i=1}^4 w_i = 1$. Table 4 shows a subset of the
370 factor weight combinations that were examined. As seen in Table 4, selecting the solution with factor scores of
371 $w_1 = 1$, $w_2 = 0$, $w_3 = 0$, and $w_4 = 0$ generates the smallest value of the performance metric in Eq. (13). Since
372 factor 1 represents the NDVI average value at the final observation, this solution suggests performing pixel
373 reassignment based solely on NDVI information with no weight applied to factors such as slope and p-value. The
374 average score function of initial arbitrary square grid solution (calculated to be 0.1339) was compared to the
375 proposed reassignment solution (calculated to be 0.0998), and yielded an improvement of 25.5%.

376 [Table 4 near here]

377 The spatial map in Fig. 5 visualizes the initial arbitrary block assignment using square grids (left) compared to the
378 final solution (right) that gave the minimum value of the performance metric in Eq. (13). The contrast in maps
379 reveals how the solution to the pixel assignment problem created natural looking clusters of differing sizes. For
380 example, along some coastline areas, clusters are long and narrow. This is intuitive because NDVI values tend to be



381 similar along the coast where many areas are comprised of sand and rock. In other areas, clusters became circular
382 and cover vast areas of known deserts in the East African regions. Small clusters also exist in the solution and, after
383 investigating, we found that many of these clusters comprise of cities and urban areas that have little vegetation. It
384 is logical that such pixels should be reassigned into the same cluster.



385

386 Figure 5 Initial arbitrary block assignment (left) compared to final solution (right).

387

388 An unbiased validation of the reassignment solution can be calculated using the average coefficient of variation for
389 the final pixel assignment and compare it to the initial square block assignment. The coefficient of variation (CV) is
390 a unit-less measure of spread that describes the amount of variability relative to the mean. CV is defined as the ratio
391 of standard deviation over the mean. Smaller values of CV indicate higher homogeneity of the clusters. The
392 average of cluster's coefficients of variations for our final pixel assignment solution is 11.762. This is a 27.4%
393 decrease compared to the average coefficient of variation for the original square blocks, which was 16.205. This is a
394 statistically significant difference in CV averages ($p=0.000529$), providing further evidence that the pixel
395 reassignment solution was able to increase the level of homogeneity within clusters. Having homogeneous clusters
396 is important when making large scale decisions about regions in East Africa that have experienced significant
397 vegetation trend changes.

398 7 Multiple Testing Implementation and Results



399 Now we can assume that the pixels in the East African region are divided into homogeneous subregions using
400 temporal assignments, as described above. Next, we summarize and apply the multiple testing procedure given in
401 Clements, et. al. (2014).

402 For notation, let m be the number of such subregions and n_i be the number of pixels/locations in the i^{th} subregion.
403 P-values at each location were calculated using a two-sided monotonic trend test at each location using the Brillinger
404 (1989) test. Specifically, we denote β_{ij} as the monotonic trend parameter as defined in the Brillinger test for the i^{th}
405 subregion and j^{th} location, where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n_i$. We also let T_{ij} and P_{ij} be, respectively, the test
406 statistic and the corresponding p-value for testing the null hypothesis $H_{ij}: \beta_{ij} = 0$ against its two-sided alternative
407 $H_{i1}: \beta_{ij} \neq 0$.

408 We apply Clements, et. al. (2014) suggestion of using a Bonferroni correction at each subregion, which combines
409 the p-values by calculating $P_i = n_i \min_{1 \leq j \leq n_i} (P_{ij})$. With H_{ij} representing the null hypothesis corresponding to P_{ij} ,
410 consider $H_i = \bigcap_{j=1}^{n_i} H_{ij}$ as the null hypothesis corresponding to i^{th} subregion. We will test the H_{ij} 's against their
411 respective two-sided alternatives and detect the direction of the alternatives for the rejected hypotheses.
412 Specifically, we apply the procedure using $\alpha=0.05$ in the following three steps:

413 *Multiple Testing Procedure Applied to Homogeneous Sub-regions:*

414 1) Apply the BH method to test H_i , $i = 1, 2, \dots, m$, based on their respective p-values P_1, P_2, \dots, P_m as follows:
415 consider the increasingly ordered versions of the P_i 's, $P_{(1)} \leq P_{(2)} \leq \dots \leq P_{(m)}$. Find $S = \max\{i: P_{(i)} \leq i\alpha/m\}$.
416 Reject the H_i 's for which the p-values are less than or equal to $P_{(S)}$, provided this maximum exists, otherwise, accept
417 all H_i .

418 2) For every i such that H_i is rejected at step 1, consider testing H_{ij} , $j = 1, 2, \dots, n_i$ based on their respective p-
419 values P_{ij} , $j = 1, 2, \dots, n_i$, as follows: reject H_{ij} if $P_{ij} \leq S\alpha/mn_i$.

420 3) For each rejected H_{ij} in step 2, decide the direction of the monotonic trend to be the same as that of
421 $\text{sign}(T_{ij})$.

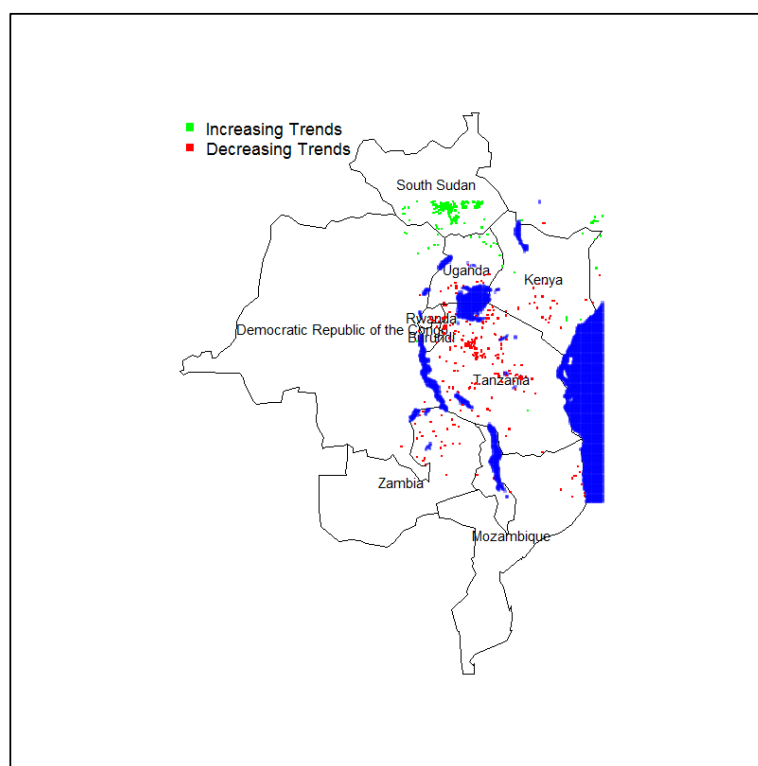
422 Step 1 and 2 identify first, the subregions and second, the locations with significant vegetation changes. The third
423 step allows one to make a more detailed analysis by identifying the directions in which these significant changes
424 have occurred. Impressively, this procedure controls the mdFDR at level α if the subregions are independent. A
425 mathematical proof of this is given in the Appendix.

426 The results of implementing this procedure to our homogenous subregions are shown in Fig. 6. Sites with a
427 significant increasing change in vegetation are plotted in green. Sites with significant negative vegetation change are
428 plotted in red. The nonsignificant sites are represented by white. Using the temporal reassignment to form



429 homogeneous subregions before implementing the multiple testing procedure detected 518 locations with significant
430 vegetation changes. Compared to the procedure in Clements, et. al. (2014) based on arbitrary square subregions,
431 this is an increase in 10 detected locations, which is indicative of a higher-powered testing procedure, while still
432 maintaining control over Type I and Type III errors.

433 Geographically, the results show increasing vegetation trends in the Northern hemisphere as well as coastal Eastern
434 Tanzania. Decreasing vegetation trends are mostly concentrated directly South of Lake Victoria. These findings are
435 consistent with historical evidence and other climate change investigations done in this region.



436

437 Figure 6 Pixels detected using the proposed heuristic reassignment solution with multiple testing procedures.

438 **8 Data Availability**

439 The data, titled "NDVI and Statistical Data for Generating Homogeneous Land Use Recommendations", may be
440 accessed through figshare. The link to the archives is: <https://figshare.com/s/ed0ba3a1b24c3cb31ebf> and the DOI is
441 https://figshare.com/articles/NDVI_and_Statistical_Data_for_Generating_Homogeneous_Land_Use_Recommendations/5897581.
442

443 **9 Conclusions and Future Research**



444 It is important to consider neighboring pixel's vegetation when making costly land management decisions that
445 would potentially relocate East African populations of people, livestock and crops. The motivation of this paper
446 stems from the opportunity to optimize the pixel assignments based on neighboring pixel data, rather than using
447 blocks in an arbitrary grid fashion, prior to using statistical methodologies to detect vegetation changes over regions
448 in East Africa. Knowing information about the homogeneous cluster to which a particular pixel belongs can provide
449 valuable insights and improved methodologies.

450 Although we demonstrated our methodology using NDVI data, the procedure can be used for any spatial-temporal
451 data, even on finer scales. Overall, by using dynamic programming to formulate a multidimensional temporal
452 assignment problem implemented by the heuristic procedure, we were able to reassign pixels to adjacent clusters
453 based on similar NDVI values over time. The results of this analysis create more homogeneous regions of East
454 Africa for decision makers to draw inferences regarding vegetation changes. We have demonstrated a powerful tool
455 for homogeneous cluster creation of pixels undergoing land-cover change using temporal satellite data.

456 Efficient land use for economic sustainability and effective land use for environmental sustainability have become
457 very important topics addressed by Cole, Et al. (2000) and Duveiller, Et al. (2007). This research may be directly
458 extended to consider additional characteristics of land and identify appropriate land use as in Usongo & Nagahuedi
459 (2008). This is especially important when considering the inclusion of multiple land purposes: residential, farm,
460 riparian borders, industrial, commercial, etc.

461 Another avenue to explore in future research is to extend the proposed methodologies to other applications of spatio-
462 temporal data. For example, monitoring and detecting transient sources in the night sky, specifically Type Ia
463 supernovae transients, is an area of astronomical research that receives much attention. Spatio-temporal astronomy
464 data has spatial dependencies that exist between pixels in astronomical images, which is well suited for a
465 multidimensional temporal reassignment to create homogenous clusters.

466 With extension of this work to other special problems, finding optimal weights will become important and relevant
467 work. Though the pixel assignment problems ultimately unveiled the appropriate weights through an iterative
468 approach, problems with extended criteria provide a greater challenge in determining appropriate or optimal
469 weights. We anticipate determination of optimal weights to be evaluated as future research as well.



470 **Appendix A**

471 **Proof.** We prove that the mdFDR is controlled at desired level α , by borrowing some ideas in Clements, et. al.
 472 (2014). Let R be the total number of H_{ij} 's that have been rejected, and V and U , respectively, be the numbers of
 473 Types I and III errors that occurred out of these R rejections. Then

$$474 \quad \text{mdFDR} = \text{FDR} + \text{dFDR} = E\left(\frac{V+U}{\max\{R,1\}}\right), \quad (A1)$$

475 since $\text{FDR} = E\left(\frac{V}{\max\{R,1\}}\right)$ and $\text{dFDR} = E\left(\frac{U}{\max\{R,1\}}\right)$ is the directional FDR. Let us consider using H_{ij} as an indicator
 476 variable with $H_{ij} = 0$ (or 1), indicating that Brillinger's null hypothesis $H_{ij}: \beta_{ij} = 0$ is true (or false). Then,

$$477 \quad V = \sum_{i=1}^m \sum_{j=1}^{n_i} I(H_{ij} = 0, P_{ij} \leq S\alpha/mn_i) \quad (A2)$$

478 where S is the number of significant subregions in the first stage of the procedure. Hence,

$$479 \quad \text{FDR} = E\left(\frac{V}{\max\{R,1\}}\right) \quad (A3)$$

$$480 \quad = \sum_{i=1}^m \sum_{j=1}^{n_i} E\left(\frac{I(H_{ij} = 0, P_{ij} \leq S\alpha/mn_i)}{\max\{R,1\}}\right) \quad (A4)$$

$$481 \quad \leq \sum_{i=1}^m \sum_{j=1}^{n_i} I(H_{ij} = 0) E\left(\frac{I\left(P_{ij} \leq \frac{S\alpha}{mn_i}\right)}{\max\{S,1\}}\right) \quad (A5)$$

482 since $R \geq S$ [borrowing the idea from Guo and Sarkar (2012)]. Let $S^{(-i)}$ be the number of significant subregions that
 483 would have been obtained if we had completely ignored the i^{th} subregion and applied the first-stage BH method to
 484 the rest of the $m - 1$ subregion p -values using the critical values $\frac{i\alpha}{m}, i = 2, 3, \dots, m$. Then, it can be shown that

$$485 \quad \frac{I\left(P_{ij} \leq S\alpha/mn_i\right)}{\max\{S,1\}} = \sum_{s=1}^m \frac{I\left(P_{ij} \leq \frac{s\alpha}{mn_i}, S = s\right)}{s} \quad (A6)$$

$$486 \quad = \sum_{s=1}^m \frac{I\left(P_{ij} \leq \frac{s\alpha}{mn_i}, S^{(-i)} = s - 1\right)}{s} \quad (A7)$$

487 Since we assume the m subregions are independent, taking expectation and inserting into FDR definition gives us



488
$$\text{FDR} \leq \sum_{i=1}^m \sum_{j=1}^{n_i} I(H_{ij} = 0) \sum_{s=1}^m \frac{1}{s} \frac{s\alpha}{mn_i} \Pr(S^{(-i)} = s - 1) \quad (\text{A8})$$

489
$$= \alpha \sum_{i=1}^m \frac{1}{mn_i} \sum_{j=1}^{n_i} I(H_{ij} = 0) \quad (\text{A9})$$

490
$$= \frac{\alpha}{m} \sum_{i=1}^m \pi_{i0} \quad (\text{A10})$$

491 where π_{i0} is the proportion of true null hypotheses among the total n_i null hypotheses in the i^{th} subregion.

492

493 We now work with the dFDR. Let $\delta_{ij} = \text{sign}(\beta_{ij})$ representing the true sign of the Brillinger's monotonic trend
 494 parameter β_{ij} j^{th} location in the i^{th} subregion and T_{ij} is the test statistic. Now, U can be expressed as follows:

495
$$U = \sum_{i=1}^m \sum_{j=1}^{n_i} I\left(H_{ij} = 1, P_{ij} \leq \frac{S\alpha}{mn_i}, T_{ij}\delta_{ij} < 0\right) \quad (\text{A11})$$

496 from which we first have

497

498
$$\text{dFDR} = E\left(\frac{U}{\max\{R, 1\}}\right) \quad (\text{A12})$$

499
$$U = \sum_{i=1}^m \sum_{j=1}^{n_i} I(H_{ij} = 1) E\left(\frac{I\left(P_{ij} \leq \frac{S\alpha}{mn_i}, T_{ij}\delta_{ij} < 0\right)}{\max\{R, 1\}}\right) \quad (\text{A13})$$

500 Making arguments similar to those used for the FDR, we then have

501
$$\text{dFDR} \leq \sum_{i=1}^m \sum_{j=1}^{n_i} I(H_{ij} = 1) \sum_{s=1}^m \frac{1}{s} \Pr\left(P_{ij} \leq \frac{s\alpha}{mn_i}, T_{ij}\delta_{ij} < 0\right) \Pr(S^{(-i)} = s - 1) \quad (\text{A14})$$

502 Notice that $P_{ij} = 2[1 - \Phi(|T_{ij}|)]$, where Φ is the cumulative distribution function of the standard normal.

503 Therefore, assuming without any loss of generality that $\beta_{ij} > 0$ when $H_{ij} = 1$, we have, for such H_{ij} ,

504
$$\Pr\left(P_{ij} \leq \frac{s\alpha}{mn_i}, T_{ij}\delta_{ij} < 0\right) \quad (\text{A15})$$



$$505 \quad = \Pr_{\beta_{ij}>0} \left(|T_{ij}| \geq F^{-1} \left(1 - \frac{s\alpha}{2mn_i} \right), T_{ij} < 0 \right) \quad (A16)$$

$$506 \quad = \Pr_{\beta_{ij}>0} \left(T_{ij} \leq -F^{-1} \left(1 - \frac{s\alpha}{2mn_i} \right) \right) \quad (A17)$$

$$507 \quad \leq \Pr_{\beta_{ij}=0} \left(T_{ij} \leq -F^{-1} \left(1 - \frac{s\alpha}{2mn_i} \right) \right) \quad (A18)$$

$$508 \quad = \frac{s\alpha}{2mn_i}.$$

509 The last inequality follows from the fact that, when $H_{ij}=1$, the distribution of T_{ij} is stochastically increasing in β_{ij} .

510 Continuing, we have

$$511 \quad \text{dFDR} \leq \frac{\alpha}{2m} \sum_{i=1}^m \frac{1}{n_i} \sum_{j=1}^{n_i} I(H_{ij} = 1) = \frac{\alpha}{2m} \sum_{i=1}^m \pi_{i1} \quad (A19)$$

512

513 where π_{i1} is the proportion of false null hypotheses among the total n_i null hypotheses in the i^{th} subregion. Thus, we
514 combine and finally prove the desired result.

$$515 \quad \text{mdFDR} \leq \frac{\alpha}{m} \sum_{i=1}^m \left(\pi_{i0} + \frac{1}{2} \pi_{i1} \right) = \frac{\alpha}{m} \sum_{i=1}^m \left(\frac{1 + \pi_{i0}}{2} \right) \quad (A20)$$

516



517 **Author Contribution**

518 Nicolle Clements completed all multiple testing evaluation and statistical analysis. Virginia Miori completed all
519 decision model development. Virginia Miori developed the heuristic approach; the heuristic was refined by
520 Virginia Miori and Nicolle Clements. The development of computer code to implement the heuristic was completed
521 by Brian Segulin.

522 **Competing Interests**

523 The authors declare that they have no conflict of interest.

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529



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